Evaluation of a GVT System for Wind Turbines

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Document No	3274/GR/01
Issue	B
Status	FINAL
Classification	Client's Discretion
Date	18th February 2003

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Revision History

Issue No:	Issue Date:	Summary
001	23/01/03	First Issue. Final
002	18/02/03	Second Issue after discussion with client Final

Circulation:	Copy No:
Gyro Energy	1
GH Glasgow	2
GH Bristol	3

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CONTENTS

1 INTRODUCTION

1.1 General

Gyro Energy Limited has requested that Garrad Hassan and Partners Ltd. (GH) undertake an evaluation of the potential of a GVT (Gyroscopic Variable Transmission) system for use in wind turbines.

1.2 Objectives

Specifically GH will aim to;

- assess the general suitability of the GVT concept of Mr Jegatheesan for use in wind turbine applications,
- identify the main advantages and disadvantages, the most appropriate application within wind technology, and the most advantageous configuration of GVT,
- appraise whether such an appropriate GVT system can be employed with net cost benefit.

1.3 Work Programme

The proposed work programme comprises the following main tasks.

1. Establish a 'baseline' conventional wind turbine design at say 1MW rated output as a basis for analytical comparisons. The 'baseline' design is variable speed with an electrical variable speed drive, has geared transmission and is pitch regulated. Representative efficiency and cost data for the baseline transmission system (gearbox, generator and electrical converter) will be generated.

2. Determine the duty specification for a GVT system that replaces the gearbox and variable speed drive of the 'baseline' turbine. The duty specification will indicate;

   - external loads in IEC critical load cases,
   - demanded design life,
   - demanded variable speed range and speed ratio range, comprising minimum and maximum input shaft speed for generator synchronous speed and options for 1000, 1500, 3000 rpm generator synchronous speed.

3. The preferred GVT arrangement for the wind turbine application will be appraised, the default situation being a direct adaptation of the existing GVT design.

4. Analysis of the preferred system. This will consider the forces in GVT components when the range of speed variation and other system properties are taken into consideration and an attempt will be made to relate this to the duty and subsequently the cost of GVT components.

   Assessment will also be made of the likely output power quality of the GVT system compared to electrical variable speed drives.

5. General engineering review of the GVT system with particular attention to life, fatigue, and wear of:

   - linear transmission elements, cams or reciprocating mechanisms
   - bearings
   - one-way clutches.

   Consideration will also be given to systems losses - bearings in general and specifically, the gyro system in respect of windage and bearing friction losses.

6. Review of cost issues. This will comprise cost breakdown data for the conventional turbine set up in comparison with the turbine with GVT. There will be near certainty about the costs of gearbox and variable speed drive avoided with a GVT transmission and the cost of a pitch system avoided by operation in stall regulation. There will probably be uncertainty about the costs of a GVT system at a reasonably mature design stage in quantity production. Costs estimates will nevertheless be attempted and at least the affordable cost of the GVT system (in order to be competitive with conventional transmissions) will be established.

7. Evaluation report. This report will document the work in items 1 to 6 and include appropriate recommendations regarding pursuing the GVT concept in wind technology. In a favourable evaluation scenario, an estimate will be provided of the content and cost of the design development and testing programme for validation of a prototype GVT system in a wind turbine.

2 ESTABLISH A 'BASELINE' CONVENTIONAL WIND TURBINE DESIGN

2.1 Baseline wind turbine summary

Rotor diameter	**56**	**m**
Rated electrical output power	**1**	**MW**
Number of blades	**3**	
Hub height	**50**	**m**
Tilt angle of rotor to horizontal	4	deg
Cone angle of rotor	-3	deg
Blade set angle	0.5	deg
Rotor overhang	2.8	m
Rotational sense of rotor, viewed from upwind	Clockwise	
Position of rotor relative to tower	Upwind	
Aerodynamic control surfaces	Pitch	
Fixed / Variable speed	Variable	
Cut in windspeed	3	m/s
Cut out windspeed	25	m/s
Gearbox ratio	**73**	
Drive train mounting	**Flexible gearbox mount**	
Gearbox mount rotational stiffness	1.8E+08	Nm/rad
Gearbox mount rotational damping	940000	Nms/rad
Gearbox casing moment of inertia	5400	kgm²
Generator model	**Variable Speed**	
Generator inertia	44	kgm²
Total Rotor Inertia	**1.13 E+06**	**kgm²**

Table 2.1.1 Summary characteristics of baseline conventional wind turbine

A conventional wind turbine of around 1 MW rated output power was considered in order to have a reference basis for comparing loads and performance of an equivalent wind turbine system with GVT transmission. Summary characteristics of the conventional system are presented in Table 2.1.1.

As part of the transmission system, the primary duty of the GVT is to transmit torque. In the approach adopted (Section 3 of this report), an operational specification for the GVT is developed considering the estimated lifetime input torque and speed history of the baseline turbine. The detailed control of the GVT will of course interact with the operational history but this is considered of secondary importance at present.

2.2 Power transmission system

Low speed shaft torque (kNm)	Loss torque (kNm)
28.4	5.40
80.8	4.80
141.5	5.70
205.9	6.20
314.9	9.40
436.4	13.10

Table 2.2.1 Mechanical losses in terms of low speed shaft torque

Table 2.2.1 indicates the mechanical losses of the conventional system – essentially, the gearbox losses.

Shaft power (kW)	Power loss (kW)
33.0	9.40
114.0	15.10
243.0	24.10
439.0	34.90
700.0	55.60
1064.0	64.00

Table 2.2.2 Electrical losses

The electrical losses are indicated in Table 2.2.2. In the conventional system with pitch control and (electrical) variable speed drive, typical full load efficiencies would be 96% for the power converter, 98% for the generator and 97% for a three stage gearbox. This implies an overall (full load) drive train efficiency of about 91%. It is essential that the efficiency of a transmission system with GVT is not less than 90% and preferable that it exceeds 91%. Energy output (directly related to efficiency) is typically about 10 times more valuable than total transmission system cost.

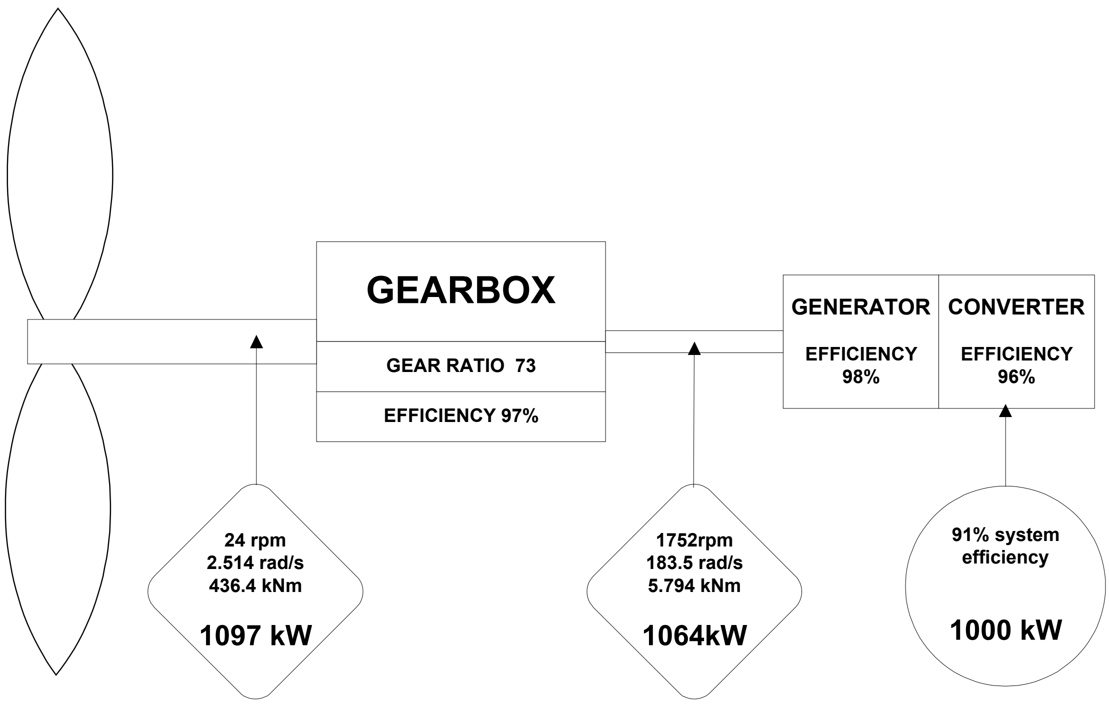


Figure 2.2.1 Typical 1MW wind turbine variable speed transmission system

If, in a GVT based transmission system, the generator efficiency remains at 98% and a single stage gearbox is employed with 99% efficiency (on the basis of 1% loss per stage of gearing), the net mechanical efficiency between input and output shaft of the GVT system must then be at least 93%. Since the GVT will only have bearing losses, it is plausible that a high mechanical efficiency above 93% can be achieved. The energy capture effectiveness of the GVT system will also, however, depend on the inherent controllability of the GVT. This is a separate issue which is addressed in Section 5.5.

In Figure 2.2.1, the layout of the representative conventional system with 1MW net electrical output at full rated power is illustrated. A corresponding system using a GVT and single stage of gearing is then developed (Figure 2.2.2).

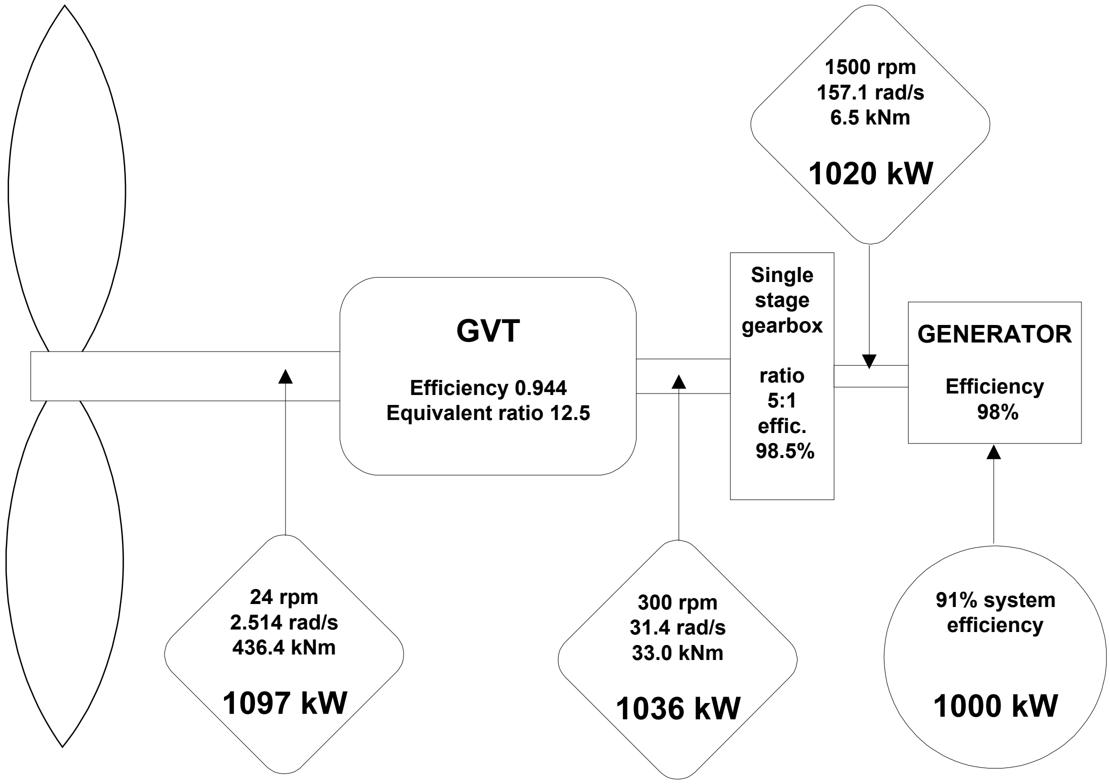


Figure 2.2.2 Equivalent system with GVT and single stage gearbox

In Section 3, the duty of the GVT transmission system on the basis of general similarity to the reference conventional wind turbine is established. Section 4 then deals with the development of a general mathematical analysis of the type of GVT system proposed by Mr Jegatheeson with an input drive oscillating the gyro axis and an output shaft motion rectified by a one way clutch system. In Section 5 GVT arrangements are summarised including the possibility of the GVT having multiple gyros and a gearbox between the wind turbine rotor and GVT. In Section 6, in addition to a general review of GVT performance characteristics, results and observations from the analytical work are coupled with engineering insights and calculations to provide an overview of the GVT system potential.

3 DETERMINE THE DUTY SPECIFICATION FOR A GVT SYSTEM

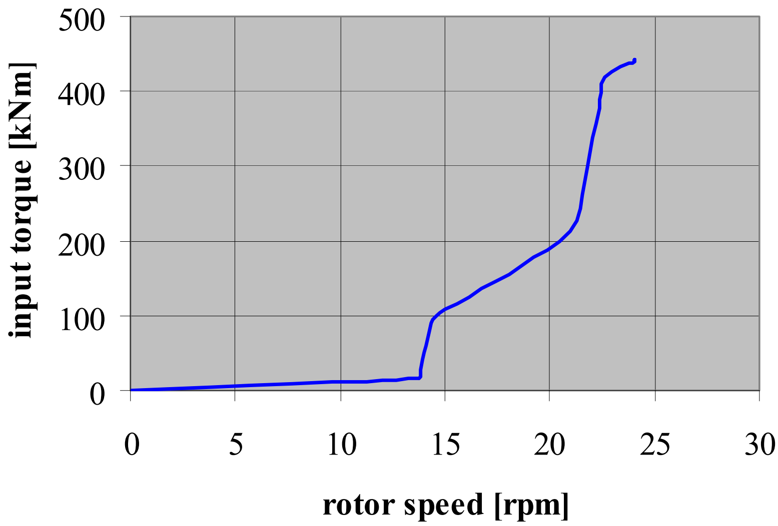


Figure 3.1 Torque speed characteristic

The operational torque/speed characteristic that the turbine controller will endeavour to track is presented in Figure 3.1. In conjunction with the gearbox torque time-at-level distribution (Figure 3.2), this gives a good description of the input torque history to the power train.

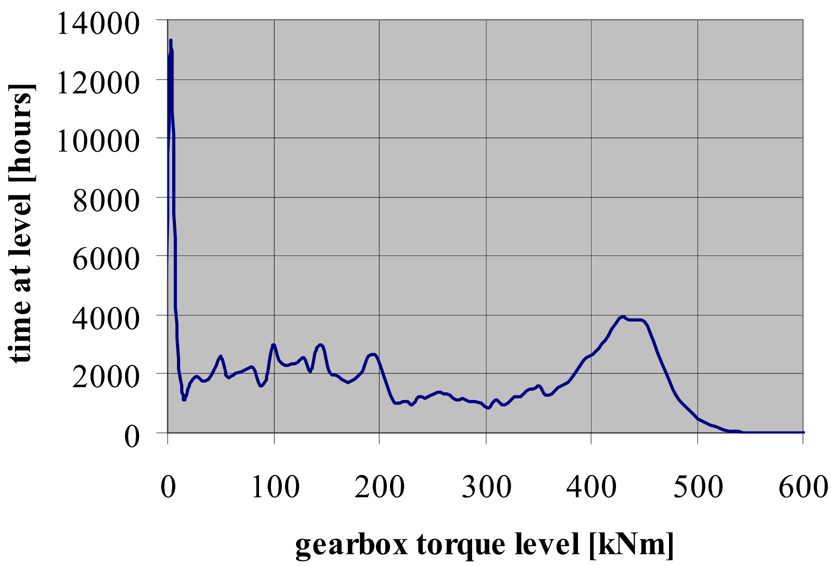


Figure 3.2 Gearbox torque – time at level distribution

The intention was to use the information of Figures 3.1 and 3.2 in GVT bearing life calculations leading to sizing and costing of suitable bearings. It emerges from the subsequent work that there are some rather more fundamental issues to address and it has not been appropriate to pursue such calculations in much detail.

4 ANALYSIS OF THE GVT SYSTEM

4.1 Background

Preliminary analysis of the GVT system was provided to GH by Mr Jegatheeson. This analysis was checked by GH and substantially extended by staff of the Control Engineering Department of Strathclyde University to derive system equations of motion and local bearing forces. The extended analysis is developed using Mathcad software. It is presented in this report as Appendices A, B and C and also provided as active Mathcad files in which equations may be modified or calculation values changed. The more extended analysis confirms the preliminary analysis developed by Mr Jegatheeson and does not conflict with it in any significant way.

The main aims of the analysis were to develop understanding of the operational characteristics of the GVT system and to be able to estimate local bearing forces with a view to evaluation of mechanical feasibility and cost.

4.2 Assumptions of the analysis and GVT system arrangement

The analysis (Appendices A, B and C) is based on the GVT arrangement proposed by Mr Jegatheeson (Figure 4.2.1) described as the "direct configuration". In the analysis presented in this report, the input motion is prescribed. Otherwise a system of differential equations would have to be solved numerically. This is quite feasible but would involve building a simulation model and is beyond the scope of the present investigation. Any input motion can be prescribed but, for simplicity, a sinusoidal translation of the sliding link in the input drive (that passes through the linear bearing in Figure 4.2.1) is assumed.

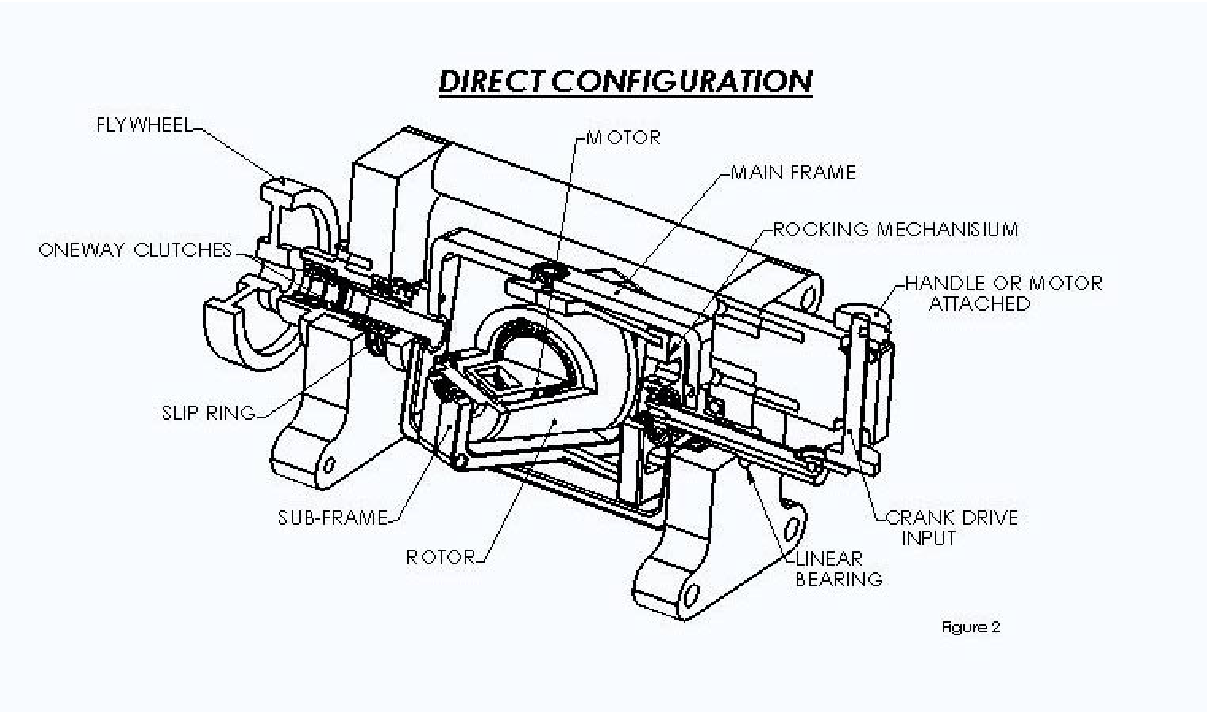


Figure 4.2.1 The GVT system modelled analytically

The output motion is also prescribed to be constant on the basis that the system will drive a synchronous generator operating at fixed speed (1500 rpm). In the main arrangement analysed (Figure 2.2.2), the GVT input speed is the low shaft rotor speed of up to 25 rpm and the output speed is fixed at 300 rpm. It is assumed that a single stage gearbox (5:1 ratio) on the output of the GVT will then provide the required generator speed of 1500 rpm. The analysis is not restricted to the system of Figure 2.2.2 and will equally describe the GVT with a gearbox ahead of its input drive or a system with no gearbox. It will be apparent that the input and output shafts of the GVT system of Figure 4.2.1 are mutually at right angles. Although the shafts are usually parallel in a wind turbine transmission, vertical generator arrangements have been considered and the issue of parallel or right angled transmission is bypassed as being quite minor in the present context.

Conceptually, the wind turbine rotor drives the "crank drive input" of Figure 4.2.1. The gyro rotates in bearings attached to the subframe and the rocking mechanism driven by the input arrangement oscillates the "sub-frame" containing the gyro about an axis with bearings attached to the "main frame". The oscillation of the gyro axis creates an oscillating torque on the output axis (axis associated with sliding input, clutches and flywheel of Figure 4.2.1) and using a one-way clutch system and flywheel beyond the clutch, the output motion is made rotary and uni-directional.

The physical elements illustrated in Figure 4.2.1 are then related to systems of axes and associated geometrical constraints determined (Appendix A) for the purposes of developing the analytical model.

4.3 Key relationships

4.3.1 Power and Gyro Angular Momentum

The analysis reveals (last part of Appendix A, "Estimation of average power transmitted") that the average power transmitted is proportional to;

- the angular momentum of the gyro,
- the input speed of the GVT,
- the output speed of the GVT.

For any given wind turbine design, the input speed range will be prescribed and the output speed set by the generator. This means that the transmitted power and input and output speeds of the GVT are also prescribed. These speeds may differ from rotor speed or generator speed of the wind turbine if there is a gearbox somewhere in the transmission path in series with the GVT but they are otherwise fixed by the system arrangement. Thus the angular momentum of the gyro is determined and there is only freedom to optimise the relative contributions from gyro inertia and gyro angular speed.

4.3.2 Gyro Bearing Loads

The GVT is directly in the transmission path and the torque on the GVT bearings as a vector parallel to the transmission axis cannot be less than the usual torque commensurate with the shaft speed and power transmitted. However the torque about an axis at right angles to the transmission axis is typically many times higher than this.

In Appendix C, under the headings "Internal Torques" and "GVT Rotor" the following equations are presented for ***Tgx(t)***, the torque about an axis parallel to the axis through the main frame bearings that connect to the rocking mechanism, and for ***Tgz(t)***, a torque that is applicable to the gyro bearings about an axis normal to the sub-frame. ***Tgz(t)*** is similar to the output shaft torque, ***Tme(t)*** (equal to it when the gyro axis is normal to the output shaft axis) and rather more easily interpreted in terms of the system of equations presented.

$$\mathbf{Tgx(t)} := (\mathbf{Igz} - \mathbf{Igy})\cdot\left[\mathbf{sin}(\theta(\mathbf{t}))\cdot\mathbf{cos}(\theta(\mathbf{t}))\cdot(\phi\mathbf{d}(\mathbf{t}))^2\right] - (\mathbf{Igy}\cdot\mathbf{cos}(\theta(\mathbf{t}))\cdot\phi\mathbf{d}(\mathbf{t})\cdot\mathbf{Nr}(\mathbf{t}) + \mathbf{Igx}\cdot\theta\mathbf{dd}(\mathbf{t})) \quad \text{.............................(1)}$$

The last term of ***Tgx(t)*** in Equation (1) is zero in steady state and the first term can be made zero by having the inertia terms ***Igz*** and ***Igy*** equal by design. Thus the remaining (middle) term dominates.

$$\mathbf{Tgz(t)} := (\mathbf{Igy} - \mathbf{Igz} - \mathbf{Igx})\cdot\mathbf{sin}(\theta(\mathbf{t}))\cdot(\phi\mathbf{d}(\mathbf{t}))\cdot(\theta\mathbf{d}(\mathbf{t})) + \mathbf{Igy}\cdot\mathbf{Nr}(\mathbf{t})\cdot(\theta\mathbf{d}(\mathbf{t})) + \mathbf{Igz}\cdot\mathbf{cos}(\theta(\mathbf{t}))\cdot\phi\mathbf{dd}(\mathbf{t}) \quad \text{.............................(2)}$$

Although the first terms of ***Tgz(t)*** and ***Tgx(t)*** cannot both be made zero by design, some study of the system with reasonable numerical values assigned shows that the middle term always dominates (steady state) in the equations for ***Tgx(t)*** and ***Tgz(t).*** In Equation (2)for ***Tgz(t)*** the middle term consists of gyro angular momentum multiplied by ***θd(t),*** the angular rate of change of the gyro axis associated with the rocking of the sub-frame within the main frame. Clearly the gyro axis angle ***θ***, is changing at the same frequency as the *input* speed. Thus ***θd(t)*** is proportional to the input speed with a factor that is related to the linkage geometry. Note now that the middle term of ***Tgx(t)*** consists of the product of gyro angular momentum and *output speed* with a further factor ***cos(θ(t))*** which will be maximum at unity and remain close to unity for comparatively small angular movements of the gyro axis.

From this it is deduced that the torque on the gyro bearings consists of two components ***Tgx(t)*** and ***Tgz(t)*** of which ***Tgx(t)*** is dominant and greater by a factor of the ratio of GVT output speed to GVT input speed multiplied by a factor associated with the link geometry. The typical values used in the example of Appendix C corresponding to the arrangement of Figure 2.2.2 with a 5:1 gearbox leads to a ratio of Tg(x) to Tg(z) of about 21 of which a factor of 12 is the equivalent gear ratio of the GVT (output to input speed ratio) and the remaining factor 21/12 = 1.75 is associated with the linkage geometry.

Unfortunately the bearing torque is not only magnified by the factor (presently 1.75) associated with the linkage (which can be optimised with the caveat that reducing this factor always makes the system physically larger). It is more significantly increased by a further factor of about 2.3 because the half-rectified waveform implies a peak torque and power much greater than average.

Thus the gyro bearings experience a compound torque. This is the resultant of two perpendicular torques, the input torque (magnified by the linkage geometry and irregularity of the waveform) and the output torque. Naturally, the input torque dominates.

In numerical terms the maximum steady state torque associated with a rated power of 1 MW and rated speed of 25 rpm is which appears on the low speed end of a conventional transmission is about 436 kNm (Figure 2.2.2) whereas a maximum torque level of ***Tgx(t)*** about 2290 kNm applies to the gyro bearings. The approximate value of 2290 kNm arises taking into consideration the result from Appendix C of a maximum resultant gyro bearing load ***Tgx(t)*** of around 1570 kNm at an output power of around 750 kW. Factoring this result up to rated power and allowing for system losses (as in Figure 2.2.2, net drive train efficiency of 0.91) give s value of the order of 2290 kNm.

In general reducing bearing loads by design involves making physically larger and heavier systems.

4.4 Gyro torque reaction

With the analysis package as presently formulated, two cases are readily examined without a system simulation involving solution of differential equations. These are;

a) a constant gyro speed is imposed with a motor providing the associated torque demand,
b) no torque is applied to the gyro axis and the gyro speed is allowed to vary.

In subsequent discussion a system is favoured where the gyro speed is not varied for control purposes and the gyros are intended to operate without externally applied torque, being motored only at start up and as necessary to compensate for bearing friction.

Note that any torques imposed on the gyro about its axis by motion of that axis in non-inertial reference frames are fundamentally oscillatory.

It is readily shown that case a) is quite impracticable. Referring to Appendix C, it is apparent that ***Tge***, the external torque on the gyro axis, is of the order of several kNm. This is a case where rigidly constant speed is imposed on the gyro. The associated power in the gyro for speeds in the range 2000 – 5000 rpm is then of the order of megawatts.

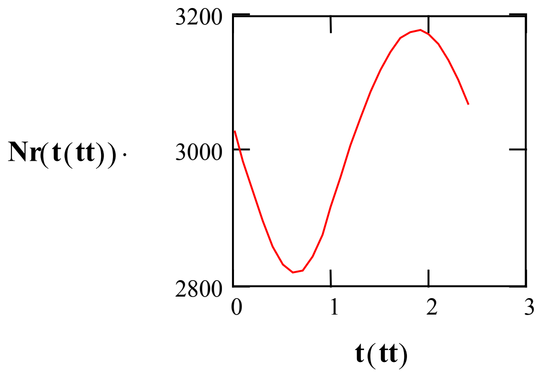


Figure 4.4.1 Gyro speed [rpm] with no applied torque

Setting the parameter λ to unity in the analysis of Appendix C engages an alternative set of equations in which there is no external reaction torque on the gyro. In that event, it would appear that the gyro speed, ***Nr,*** (Figure 4.4.1, ***t(tt)*** is time in seconds) will oscillate at the frequency of the input (i.e. wind turbine rotor frequency) with about 6% variation. A servo-controlled motor with rating of the order of a few kW, as is determined necessary to make up losses in the hydrodynamic bearings of the gyro, will be employed. This is quite feasible and providing the rating is low, it will be of little consequence whether a DC motor with inverter or induction motor is employed.

As the system experiences large oscillatory loads in all the bearings and linkages, the 6% variation in gyro speed will have little consequence.

5 GENERAL ENGINEERING REVIEW OF THE GVT

5.1 System components

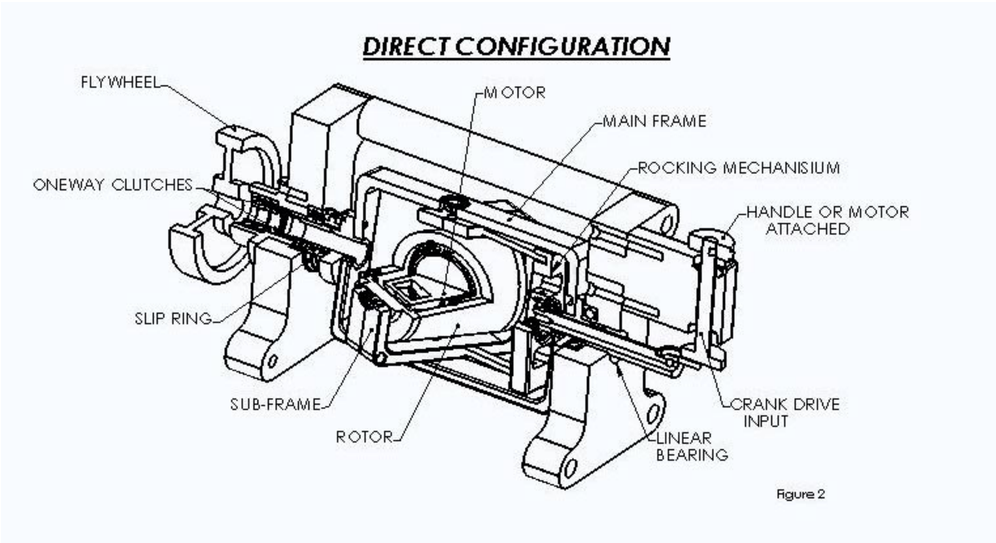


Figure 5.1.1 Model of direct configuration

Figure 5.1.1 (identical to Figure 4.2.1) shows once more the direct drive arrangement from which a list of key components is evident.

- Reciprocating crank or cam driven input drive
- Rocking mechanism comprising a linear sliding element and link arm connecting to the gyro sub-frame
- Sub-frame containing gyro rotor (with internal electric motor) and gyro bearings
- Main frame cage connected to the output shaft with sub-frame bearings
- Power slip ring on the output shaft providing a power supply to the gyro
- One way clutches on the output shaft to prevent back drive from the oscillating output torque
- Flywheel on the output shaft to smooth output rotation.

It should be stressed that Figure 5.1.1 shows a demonstration model (which has been manufactured and is operational) but it is not directly a prototype design suitable for a wind turbine. Many details and possibly some major features may need to change in an appropriate design of a prototype GVT for testing in a wind turbine.

5.2 Gyro bearing design

It is evident from the analysis of Section 4.3.2 based on the results of Appendix C, that the most critical loads are on the gyro bearings.

Consider now an example calculation directed at selection of a rolling element bearing.

A radial bearing load of 3000 kN (assuming 0.5 m spacing between the bearings in a system with one GVT only) is assumed for a design life of 25 years with 17 years of continuous operation.

Note:

- In the systems to be recommended with multiple GVTs, the bearing *torque* will reduce by a factor of ***n*** in a system of ***n*** GVTs but the bearing *force* will only reduce by such a factor if the bearing separation distance is *not* reduced. The bearing spacing of more than 0.5 m can be considered but this will then almost certainly be more than is needed for the gyro rotor size (as determined by the demanded gyro inertia and accommodation of the gyro motor). Also the arrangement indicated in Section 6 (Figure 6.6.1) will become impracticable if the GVT units become too bulky.

- The assumed load of 3000 kN is less than the bearing load at rated power when the torque of 2290 kNm at 0.5 m bearing separation would give rise to a load of 4580 kN.

Basic Life Equation

$$L_{10} = \left(\frac{C}{P}\right)^p$$

where L_{10} = Revolutions x 10^6

C = Basic Load Rating

P = Equivalent Dynamic Load

p = Exponent – 3 for ball bearing = 3.33 for roller bearings

Assume speed = 2000 rpm

Assume bearing distance is 0.5m, and load = 3000 kN

Required life = 2000 x 60 x 8760 x 17 = 1.78 x 10^{10} revolutions

Therefore, required L_{10} = 1.78 x 10^4

Required basic load rating = 3000 x 10^3 x $(1.78 \times 10^4)^{1/3.33}$ = 56.52 x 10^6N

This is far too high for an acceptable bearing selection and discussion with other engineering consultants has highlighted that when bearings are fatigue critical, there is every likelihood that the shaft design may be equally or more problematic.

Whilst bearing loads have not been optimised and may be reduced with some detailed design effort, it is clear that the gyro bearings are well out of the range of standard rolling element bearings and hydrodynamic bearings must be used. They will be special developments (not unusual but equally not off-the-shelf) yet not necessarily expensive in production as they are much simpler in terms of precision engineering components than rolling element bearings.

5.3 Other components

Referring to the indicative results of Appendix C, the reciprocating drive is highly loaded as is to be expected of the input system. There is however reasonable freedom to engineer this part of the system and hence no fundamental concern about it. The one–way clutch design could be very important in a final realisation of the system. The design is eased by the moderate output speed of the GVT, at least in the arrangement with a 5:1 gearbox as in Figure 2.2.2.

Mr Jegatheeson has special patented solutions for the one-way clutches. The version of the innovative clutch design most applicable to the GVT is a design with ratchet and pawl in a "vernier" arrangement. Thus with say N engaging elements on one side mating with N+1 on the other, there is only one point of engagement (as is the norm for a pair of meshed gear wheels) but backlash impacts are greatly reduced and frictional wear is avoided. The clutch is of interest (as several variants in the patent application) as an engineering item in its own right. The concepts seem very promising but will need more detailed evaluation to be confirmed as a solution in the GVT system.

5.4 Power quality

In the arrangement of Figure 2.2.2, unless provision is made to smooth the torque in the gearbox (e.g. multiple input shafts from several GVTs), the input torque to the generator will appear as a half-rectified wave at the frequency of the main wind turbine rotor. This may not be acceptable and has the following definite disadvantages;

- The peak power and peak torque will be $2\sqrt{2}$ times nominal and will involve a similar factor on generator or gearbox cost depending on where and how the power is smoothed. (The factor of $2\sqrt{2}$ is purely theoretical and applies to a half rectified sine wave. In fact, the proposed arrangement is more favourable and the factor reduces from $2\sqrt{2} = 2.83$ to about 2.3. This nevertheless remains a significant magnification on power and loads which will always affect the cost of some part of the transmission system).
- The fluctuating nature of the output torque may impose additional fatigue loads.
- If the full toque variation is presented to the generator, the torque cycles will be at a frequency where flicker problems may arise. There may also be an issue of higher frequency harmonic pollution. It would defeat the objective of having the GVT system to use additional power electronics to solve such problems.

The poor output waveform is a major concern and would not be acceptable. It is not a fundamental feasibility problem but rather that the GVT would be quite unattractive to any potential user (wind turbine manufacturer) without improved output power quality.

A reasonable solution would appear to be to have a number of GVTs in parallel providing a number of input shafts to the single stage gearbox. The wave form irregularity factor (ratio of peak to mean) of 2.3 would imply more than doubling of the gearbox cost compared to a conventional single stage gearbox design. In addition, the gearbox design may not be straightforward since a step up in gear ratio is required (when a step down would better suit the multiple input shafts). However the GVT has effectively replaced the two most expensive stages of gearing and the special single stage gearbox after the GVT may well be affordable.

An alternative is to have say 3 GVTs, each output to a separate (simple) single stage gearbox and generator. The 3 electrical outputs would then be combined electrically to give smooth power to the grid. Each generator would have to be rated mechanically for over twice the average torque corresponding to rated power but not thermally rated by as large a factor. Again there is a significant but possibly affordable cost penalty.

5.5 Control characteristics

$$P_{AV} \propto \Omega . N_R \quad ; \quad T_{GVT} \propto N_R$$

where

P_{AV} - average power transmitted by GVT
T_{GVT} - average torque transmitted by GVT
Ω - turbine rotor speed
N_R -GVT rotor speed

$$T_R \propto \Omega^2$$

where

T_R I -turbine rotor torque
Ω - turbine rotor speed

In steady state $T_{GVT}=T_R$. Hence, during variable speed operation,

$$N_R \propto \Omega^2$$

and

$$E_S \propto \Omega^4$$

where

E_S - energy stored in GVT gyro

So to effect a change in turbine rotor speed from $0.7\Omega_R$ to Ω_R requires a change in stored energy of 75% (taking 0.7^4 as approximately ¼) of energy stored at Ω_R; that is, assuming the usual parameter values,

$$\tfrac{3}{8} I_{gy} N_R^2 = 9.4\ MJ$$

With 10kW motor, this would take 940 seconds. The response time can be reduced substantially if a gearbox is placed between wind turbine rotor and GVT unit i.e. the input speed of the GVT is increased. This is not a desirable solution, however, for reasons discussed in Section 6.2. These long response times have consequence that T_R will not be balanced for long periods by the reaction torque from the GVT and there have to be reliance on pitch regulation to prevent overspeed even in below rated wind conditions with an accompanying loss of aerodynamic efficiency.

In order to reduce the response time to an acceptable level for good control, i.e. of the order of a few seconds, multiple gyros are required and the net rating of the gyro motors must increase. In view of the associated power consumption the operation would need to be regenerative and coupled to the system electrical output.

In recognition of this potentially major problem of slow torque reaction, Mr Jegathesson has proposed a much more effective way of regulating torque. This involves changing the link geometry controlling the range of angular movement of the gyro axis.

Controlling torque in this way can be understood by reference to Equation (2) of Section 4.3.2. The dominant middle term of that equation consists of the gyro angular momentum multiplied by $\theta d(t)$, the angular rate of change of the gyro axis associated with the rocking of the sub-frame within the main frame. Since the frequency of movement of the gyro axis is determined by the input shaft rotation, changing the amplitude of the angular movement of the gyro axis also changes the rate of movement of the gyro axis which, according to Equation (2), alters the torque reaction.

The average torque reaction per cycle depends on the range of angular movement (which as has been explained directly affects the rate of angular movement) but it is also affected by the position of the gyro axis. For example oscillating the gyro axis over a range of θ around 0° will produce almost no torque reaction compared to oscillating over the same range of θ around 90°. The one-way clutch system creates in each cycle of input shaft rotation an output power stroke and reaction stroke. The link geometry can be changed during the reaction stroke avoiding operation against high forces.

Thus the torque reaction can be controlled by varying link geometry in order to change the range of oscillation of the gyro axis or the mean position of oscillation. Furthermore, the control action can take place during the reaction stroke, the power stroke or both but perhaps preferably during the reaction stroke.

Variable link geometry introduces additional complexity and a further system component, possibly a servo controlled hydraulic ram as one of the link components. Such a ram would, however, potentially be well suited to the input regime of high forces, comparatively small displacements and demand for fast response. Thus in many respects this seems a good solution.

The capability to modify the torque reaction via the linkage also suggests that the gyros could be run at constant speed with almost negligible power demand. It is unfortunately at too late a stage in the present study to allow a significant investigation of a GVT system with variable link geometry but this feature is probably an essential part of a viable GVT system for a wind turbine. Otherwise the problem of rapid control of torque has no clearly satisfactory solution.

5.6 General operational and electrical design issues

There is likely to be a substantial amount of stored energy in the gyro(s) commensurate with the stored energy in the wind turbine rotor. In the specific example of Appendix C, the stored energy in the gyro, corresponding to a rotational inertia of 100 kgm^2 at 5000 rpm is 2.5×10^7 J. This is greater by a factor of 3.5 than the energy stored in the wind turbine rotor, 3.6×10^6, corresponding to an inertia of 1.13×10^6 kgm^2 at 25 rpm.

As has been discussed in Section 4.3.1, the design must maintain the angular momentum of the gyro and even at the much reduced gyro rotational speed of 2000 rpm and with the required associated inertia of 239 kgm^2, the energy stored in the gyro is still 50% more than the energy in the wind turbine rotor.

The gyro inertia is not of course in series with the wind turbine rotor inertia. As has been discussed, it affects the capability of the gyro to change speed and influences the torque reaction on the wind turbine rotor. The inherently high gyro inertia implies a slow response and is considered to rule out variation of gyro speed as a primary control method.

Systems in which the gyros are part of the power take-off system and exchange energy regeneratively are conceivable. The energy storage in the gyros may then be useful for frequency response and reserve if this energy is controllable. With power take off in the gyros, they could be used to maintain torque reaction during grid faults. It is beyond the scope of the present evaluation to investigate such designs.

6 PREFERRED GVT ARRANGEMENT

6.1 Arrangements under consideration

The GVT arrangements reviewed are restricted to those embodying the system proposed by Mr Jegatheeson. The essential features of this are a reciprocating drive input that oscillates the gyro axis and an oscillating output torque which is rectified by a one way clutch system.

The arrangements to be considered are in two main classes involving;

- a gearbox before or after the GVT
- one or more gyros in the GVT operating in parallel with different phase angles to the input

6.2 Effect of gearbox position

As with a gearbox, the input torque plays a major role in the size and cost of the GVT unit. Compared to a conventional electrical system, the GVT is a relatively poor quality variable speed drive with slow response (if dependent on gyro speed) and no control of harmonics (irregular waveform). It would therefore follow that for the GVT to be viable, it must replace the gearbox or most of the gearing.

It seems reasonable to consider a single stage of gearing after the GVT when it will be a low proportion of system costs but much more questionable to consider a gearbox ahead of the GVT. Ahead of the GVT, it would be a major cost and is likely to substantially erode the potential cost benefit of a GVT system. There is little point in the GVT becoming a small, light weight, low cost variable speed drive as it will not compete with its electrical equivalent in performance and also may not on cost.

Thus to have economic potential in a wind turbine system, it is important that the GVT substantially replaces a gearbox. It is also desirable for the general design of the GVT that the output speed is not too high. Main thrust bearing losses will increase with output speed as will the speed range of the main frame. It may be possible to dispense with any gearbox but, provisionally, the arrangement of Figure 2.2.2 with single stage of gearing after the GVT seems the most promising.

6.3 Variable link geometry

Most of the present study has focussed on a system with fixed link geometry. It was at a late stage in the project that the control response issue was highlighted as a major concern. Mr Jegatheeson then proposed the variable link geometry solution. This solution is now seen as both necessary and desirable.

6.4 Multiple GVTs

The output power quality of a GVT unit with single gyro is deemed to be inadequate and this has led to the view that there must be two or more gyros in parallel. This leads to a little complication in providing an input drive to a number of units but perhaps no more so than in the multi-cylinder arrangements that are universal in motor vehicles. There may be more of a complication is in combining the outputs. Some discussion with gearbox specialists should clarify this. The mechanical options therefore need review while the electrical methods are straightforward but more than double the net generator peak power rating.

6.5 Rotor braking and safety

In the GVT system, the input and output shafts are not torsionally connected by any physical link. Input and output rotations do not communicate unless the gyro rotates. Thus a mechanical parking brake anywhere on the output, high speed end is ineffective. However, the control of torque reaction by variable link geometry implies high capacity actuators on the low speed end capable of feeding back torque reaction to the wind turbine rotor. It would seem that this is the logical way to provide mechanical braking as back up to the pitch system (which is expected to be of the current mainstream type for large wind turbines with independent pitch drives on each blade). Otherwise a parking brake is required on the low speed shaft of the wind turbine and this will be large and expensive.

6.6 Proposed GVT arrangement

The proposed arrangement is presented in Figure 6.6.1. It embodies multiple GVT units and each unit has variable link geometry, the length of the input slides being controlled by hydraulic rams. This arrangement has been developed in the last days of the project and is not to be taken as anything other than a plausible arrangement that looks to be feasible in principle.

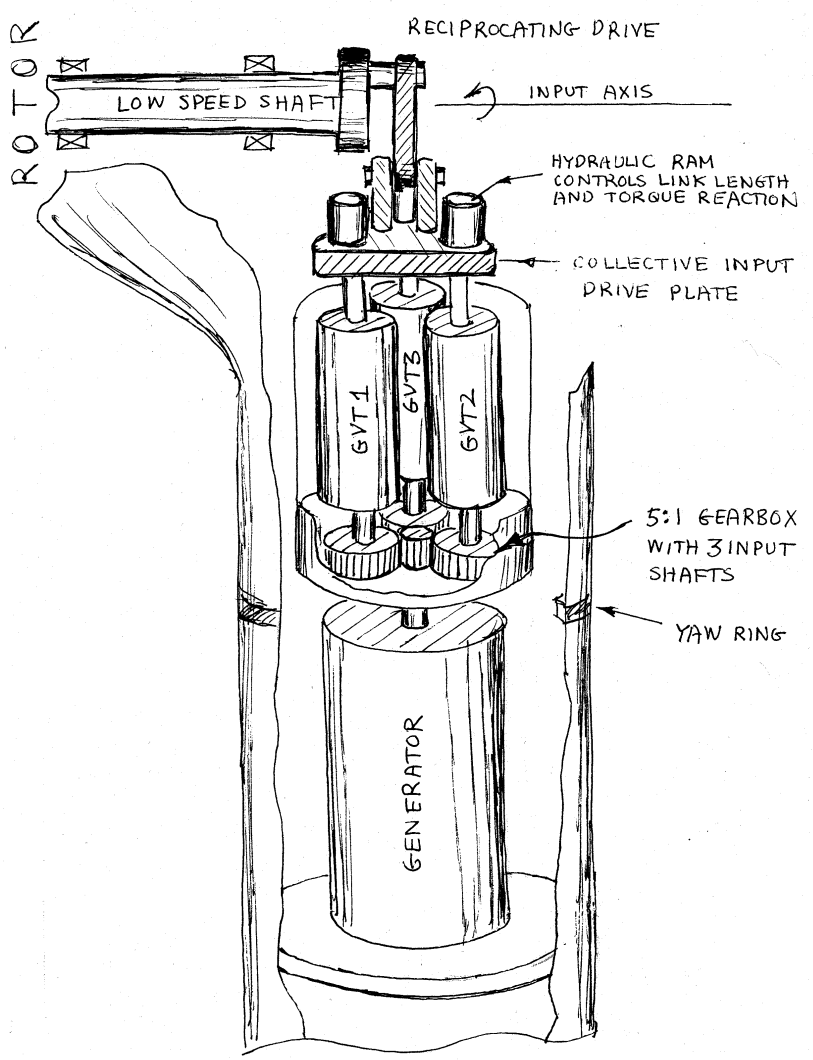


Figure 6.6.1 Preliminary layout of a GVT transmission for a wind turbine

In Figure 6.6.1, the three gyro inputs are each 120° different in phase, in terms of the rocking motion of the gyro sub-frames (internal to the GVT units and not shown). Conceptually this is controlled by the phase of the strokes of the three rams and maintained as the rams change amplitude of stroke to regulate torque reaction. It is recognised that this arrangement may not be optimum. It may be better for example to hardwire the phase difference into the connection from the low speed shaft as in a "three cylinder engine". There are many issues for further consideration.

The first step will be to fully consolidate a view of the necessary dimensions of the GVT units. The preliminary designs at 3000 rpm gyro speed suggest that the gyro rotor (as a hollow steel cylinder in a set of three) might be about 0.5 m diameter and also about 0.5 m length. Gyro rotor design will depend on the accommodation of a motor and dimensional constraints associated with that and final outer casing dimensions will depend on main frame, sub-frame and linkage design.

The system may then be realised as a complete cylindrical unit of GVTs, gearbox and generator or, depending on finalised GVT dimensional requirements, the GVT module may become more of a disc shape and perhaps of larger diameter.

It may be better to mount the actuators that control link length directly on the GVT units and have thrust capability but rotational freedom in the connection to the collective input plate (which in turn may end up as a piston head in a cylinder). The illustration of Figure 6.6.1 is purely conceptual at this stage.

The vertical generator arrangement is not usual in present generation wind turbines. However, such arrangements have been employed and the associated design issues are familiar to wind turbine designers. These include provision of auxiliary power to the nacelle via a low speed slip ring (say 0.25 rpm design yaw rate) and toleration of yaw motion as superposed on the rotation of the generator drive shaft.

7 REVIEW OF COST ISSUES

7.1 Allowable costs

Some significant distinction must be made between the actual manufacturing cost of a component i.e. the cost to the component supplier and the cost to the wind turbine manufacturer of the component which is in effect the price of the component. The wind turbine manufacturer will then sell on the complete wind turbine system with a further mark up constituting the price of the wind turbine.

Conventional wind turbine system			
	Price fraction	**Price [£/kW]**	**Price for 1 MW [£]**
Blades	0.212	80.56	80560
Hub	0.028	10.64	10640
Gearbox	0.162	61.56	61560
Rotor bearings	0.050	19.00	19000
Generator	0.106	40.28	40280
Nacelle	0.088	33.44	33440
Yaw	0.027	10.26	10260
Variable speed system	0.106	40.28	40280
Pitch system	0.088	33.44	33440
Tower	0.133	50.54	50540
Total turbine	**1.000**	**380.0**	**380000**

Table 7.1.1 Price make up of a conventional wind turbine system

System with GVT and single stage gearbox			
	Price fraction	**Price [£/kW]**	**Price for 1 MW [£]**
Blades	0.212	80.56	80560
Hub	0.028	10.64	10640
Gearbox (single stage)	**0.061**	**23.26**	**8840**
Rotor bearings	0.050	19.00	19000
Generator	0.106	40.28	40280
Nacelle	0.088	33.44	33440
Yaw	0.027	10.26	10260
GVT	**0.207**	**78.58**	**78577**
Pitch system	0.088	33.44	33440
Tower	0.133	50.54	50540
Total turbine	**1.000**	**380.000**	**380000**

Table 7.1.2 Price make up of a system with GVT and single stage gearbox

The data of Table 7.1.1 is based on a price split of bought-in components typical of large commercial (land based) wind turbines employing active pitch control in combination with an electrical variable speed system. It can be seen that the combined price of gearbox and variable speed system amounts to almost 27% of total turbine price. In the present context price includes delivery and installation.

Gear costs are estimated on the basis of cost/weight. It is considered that the low speed part of a 3 stage gearbox will weigh approximately 75% of the total (including shrink disc, etc.). The 1 MW wind turbine under consideration has an input speed of 25 rpm and the transmission system connects to a generator running at 1500 rpm. Thus a gearbox ratio of 60 is required. If there is a single stage gearbox after the GVT with gear ratio of 5, the GVT must provide the equivalent of a gear ratio of 12. The single stage gearbox will then have an input torque reduced by a factor of 12 compared to the wind turbine rotor. The GVT input torque is further reduced by a factor of about 1/0.75 (the high torque input stage of a three stage gearbox is considered to comprise 75% of weight and cost) compared to a three stage gearbox. However, the irregular half-rectified input waveform will increase design torque levels by a factor around 2.3 according to the present design of GVT.

Thus the expected cost [£] of the single stage gearbox is:

$$C_{gb} = \frac{61560 \times 0.75 \times 2.3}{12} = 8840$$

For a megawatt scale system, the data of Figure 7.1.2 implies that the GVT and its associated single stage planetary gearbox is selling at around £87,400 and, in mature production, allowing about 20% for all mark-ups should cost less than about £73,000. The costs presented in Tables 7.1.1 and 7.1.2 are up to date and highly competitive. On that basis the effective budget of £73,000 for a megawatt scale complete GVT transmission system (including singe stage gearbox) should not be unduly optimistic.

7.2 GVT costs

There are basic problems in attempting to directly derive GVT costs. The investigation started with a model (essentially realisation of conceptual design) rather than a prototype design. It is now apparent that any prototype that is likely to be satisfactory will have multiple gyros each with variable geometry linkages. Neither the variable geometry links nor the means of dealing with multiple inputs and outputs has been engineered. It is not sensible to attempt to cost these items at this stage.

The conventional megawatt scale wind turbine might employ a gearbox of mass around 8 tonne and generator of about 4.5 tonne. The generator is essentially the same in the GVT transmission system. Well established scaling rules for gearbox design suggest that a single stage gearbox (Figure 6.6.1) appropriately sized for the net input torque will weight around 0.5 tonne (maybe allow up to 1 tonne as the three input shaft arrangement is unusual?). There is about 3 tonnes of essential mass in the GVT rotors and it remains to determine the total mass of a GVT system with casing, frames, linkages, actuators, motors, clutches and flywheels. There is then a budget of about 4 tonnes for completion of the GVT system if it is to stay within the mass of a conventional gearbox.

As a mixture of electrical, hydraulic and structural components, it is very difficult to say whether, it should cost more or less per kg than a gearbox. The cost estimates of Section 7.1 imply that in replacing the dual function of gearbox and variable speed drive the GVT transmission can be about 17% more expensive than a gearbox. That of course only gives economic parity and the GVT system as an innovation must show some significant cost of energy (COE) advantage. Such advantage could, however, arise in improved in efficiency as much as in capital cost.

For example, the losses in a gearbox gearing are usually considered to be 1% per stage. The generator efficiency for conventional or GVT system is taken as 98% (Figure 2.2.1). There is nothing in the GVT system that should intrinsically have high losses. With 10 kW input to the three gyro rotors, even as a continuous demand (and this is not expected but of course no assessment of the hydrodynamic bearing losses or power demand has been made), the loss would be 1%. Thus if the GVT could be made with overall losses within 2% of rated power capacity, it would have a system efficiency of around 95% and this would 2 – 4% better than the norm for large variable speed wind turbines. Such small efficiency gains may sound unimpressive but in terms of cost of energy, they directly affect energy production and each percent of efficiency has a percentage value of about 10 times relative to the capital cost of the transmission system components under consideration.

The effective efficiency with regard to energy capture is not however, purely a matter of system electrical and mechanical losses. Maximising energy capture, especially in wide range variable speed operation, depends critically on the effectiveness of the wind turbine control system and a final view of the GVT transmission impact on COE will only come when its control characteristics are well understood. If, however, the control of torque reaction via the input linkage is effective in providing reaction torque changes with lags of the order of a few seconds or less, then there is little apparent reason that the GVT system would not match the conventional system in controllability.

Reliability directly affects energy production and is a primary concern with wind turbine transmission systems. Electrical drives and gearboxes have figured very significantly in operational failures.

With hydrodynamic bearings, with no gear teeth on the input end of the transmission and a simple single stage gearbox on the output, the system has potential for very low wear. The clutch system needs further investigation to get a clearer view of reliability issues. At the highly loaded input crank end the speed of rotation will be too low for hydrodynamic bearings but a hydrostatic bearing solution should be feasible.

8 CONCLUSIONS

8.1 General Conclusions

The GVT has potential for wind turbine applications. To achieve parity with a conventional transmission system at 1 MW scale, there is an ample efficiency budget of up to about 7% losses in the GVT system and a substantial capital budget of about £73,000 as the GVT system cost in mature production. However, nothing conclusive about GVT system efficiency or cost has been determined in the present work.

Considerably more work is necessary to appraise general arrangement issues and specific issues especially relating to loads and control that will all affect dimensions and costs.

The major finding of the present work is that fast torque reaction via variable input link geometry and multiple gyros will be essential.

8.2 Specific conclusions

- A 1 MW scale wind turbine with GVT transmission system has been evaluated in comparison to a conventional system as baseline. The transmission system lifetime torque/speed history of the conventional turbine is represented by data derived from simulations.

- In the current preferred arrangement the low speed shaft is input to the GVT and a single stage (5:1 ratio) gearbox is connected to the GVT output.

- An analysis of the GVT arrangement proposed by Mr Jegatheeson has been developed in sufficient detail to define component loading and appraise operational characteristics.

- It is shown that the angular momentum of the gyro(s) in the GVT system is uniquely determined by the power transmission level when the input and output speeds have been set.

- Gyro bearing loads are critical in GVT design. These gyro bearings experience radial forces associated with a torque which is greater than the input shaft torque by a factor (of the order of 4) which depends on the linkage geometry and output wave form.

- Hydrodynamic bearings will be required for the GVT.

- It appears to be impractical to regulate rotor reaction torque by variation of gyro speed. It is shown that the gyro(s) in the GVT system may typically have more stored energy than the wind turbine rotor and that, consequently, torque control based on gyro speed would involve long time constants and be ineffective.

- Fast regulation of rotor torque reaction is available from conventional electrical variable speed drive systems. It is considered that a GVT system replacing gearing stages and an electrical variable speed drive must also provide fast control of wind turbine rotor torque reaction (time constant of a few seconds or less).

- Control of rotor torque reaction via variable input link geometry is considered a promising and essential feature of GVT system design.

- It is then considered that the gyros should operate at essentially constant speed. Small cyclic speed variations perhaps in a range up to 10% will be imposed by inertial interactions as the gyro axis moves in rotating reference frames. Resisting such speed variation would lead to quite unacceptable power requirements for the gyro rotors. Thus the gyro motor system should be designed to make up bearing loss but definitely not to hold gyro speed synchronously.

- The half-rectified output power waveform of a GVT unit with single gyro leads to unacceptable power quality. Thus a system with multiple gyros is considered to be essential.

- It is probable that the variable link geometry should be designed in such a way as to be capable of providing rotor-braking capability. This should not be a problem in terms of load capacity as the system is on the input (low speed) end of the GVT and will see linear forces commensurate with over twice rated torque (owing to the half rectified waveform associated with the one way clutch transmission).

- A conceptual GVT arrangement with 3 gyros and variable link geometry is illustrated. No details of the link geometry have been addressed.

- Affordable cost estimates for the GVT system have been derived. At MW scale the complete GVT transmission system between low speed shaft and generator should cost less than about £73,000 in volume production or have compensating efficiency benefits.

- Each % of efficiency is worth about 10% in transmission system costs i.e. around £7,500 of capital cost. It would appear that the GVT has some potential to be more efficient than the equivalent conventional system, but a lot of design and development will be necessary to prove efficiency benefits.

- Reliability of the GVT system will depend on how well the innovative clutch system of Mr Jegatheeson can perform but the design is promising and is likely to be effective.

9 RECOMMENDATIONS

The GVT system has potential for efficiency gain and good allowable cost margins to permit net cost benefit over conventional transmission systems. However, especially with several "make or break" issues appearing at a late stage in the present evaluation, the appraisal cannot be more definite.

An indication is provided of the recommended steps to establish GVT transmission systems in the wind turbine market. The main phases are outlined. Each phase would end with a milestone review appraising whether to stop or go further. At present, it is recommended to proceed to Phase 2. Phase 2 needs to be completed with a positive outcome before there is much likelihood of a wind turbine manufacturer taking an interest in the transmission.

Government funding in Europe or other world areas may be helpful in the development phase. The cost presented for all phases beyond Phase 2 are very approximate. There is nevertheless a good basis for the development costs suggested here for all phases except for Phase 6 where strategic decisions whether to seek partnership, set up new manufacturing facilities or purely license technology will have a crucial influence on costs.

Phase	Task	Cost
1	Preliminary Evaluation of GVT (present work – completed)	£16,000
2	Outline design of GVT transmission	£65,000
3	GVT prototype (detailed) design	£250,000
4	Prototype manufacture (MW scale)	£280,000
5	Wind turbine field test	£250,000
	Total for GVT demonstration	**£861,000**
6	Development of a production unit	£4,000,000

Table 9.1 Indicative development costs of GVT systems for wind turbines

The next stage of work, Phase 2, would comprise the following.

Phase 2	Outline design of GVT transmission
a	Conceptual designs for variable linkage geometry
b	General arrangement issues: gears, input drive design, numbers of gyros etc.
c	Transmission system simulation model including control system design
d	Load specification of GVT system components
e	Outline design/selection of main components confirming dimensions and layout
f	Component and system mass and cost estimates
g	Report and milestone review

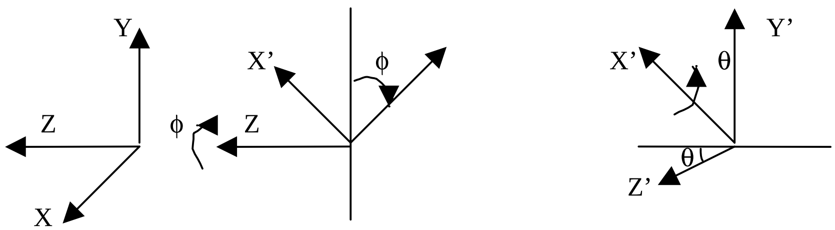
Table 9.2 Content of next phase of proposed work

APPENDIX A

AXES DEFINITION, LINK ARM GEOMETRY, EQUATIONS OF MOTION AND AVERAGE POWER

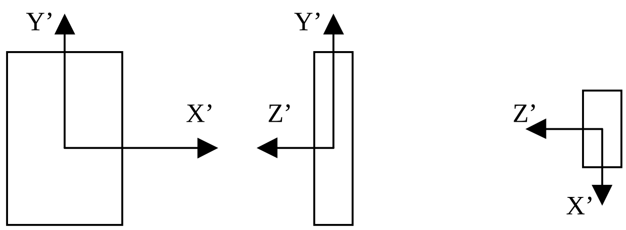
Choice of Axes for GVT Rotor and Subframe:

x y z axes fixed with respect to earth, with Oz along the main shaft and Oy vertical.



rotation about z axis　　　rotation about x' axis

x' y' z' axes fixed with respect to subframe, as shown below. When $\phi=\theta=0$, x, y, z axes and x', y', z' axes coincide.



Front　　　Side　　　Top

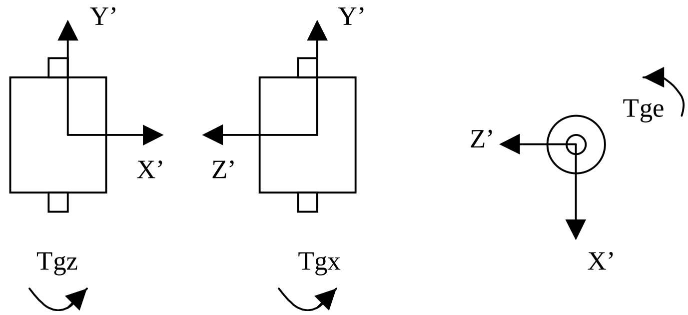
Angular velocity of subframe in x', y', z' axes:

$$\omega s(t) = \begin{pmatrix} \frac{d}{dt}\theta(t) \\ \sin(\theta(t))\cdot\frac{d}{dt}\phi(t) \\ \cos(\theta(t))\cdot\frac{d}{dt}\phi(t) \end{pmatrix}$$

Angular velocity of gvt rotor in subframe axes:

$$\omega g(t) = \begin{bmatrix} \frac{d}{dt}\theta(t) \\ \left(Nr(t) + \sin(\theta(t))\cdot\frac{d}{dt}\phi(t)\right) \\ \cos(\theta(t))\cdot\frac{d}{dt}\phi(t) \end{bmatrix}$$

GVT Rotor



Angular momentum of gvt rotor in subframe axes: $$Lg(t) = \begin{bmatrix} Igx \cdot \frac{d}{dt}\theta(t) \\ Igy \cdot \left(Nr(t) + \sin(\theta(t)) \cdot \frac{d}{dt}\phi(t) \right) \\ Igz \cdot \cos(\theta(t)) \cdot \frac{d}{dt}\phi(t) \end{bmatrix}$$

Torque acting on the gvt rotor: $$Tg = \begin{pmatrix} Tgx \\ Tge \\ Tgz \end{pmatrix}$$

Tgx and Tgz stem from bearings, while Tge is driving torque for gvt rotor, (Tge means External Torque).

In general, $$Tg(t) = \frac{d}{dt}Lg(t) + \omega s(t) \times Lg(t)$$

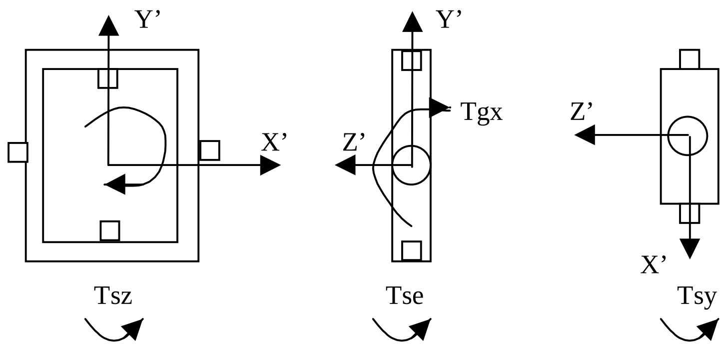
So : Equations of Motion for GVT Rotor

$$Tgx(t) = (Igz - Igy) \cdot \sin(\theta(t)) \cdot \cos(\theta(t)) \cdot \left(\frac{d}{dt}\phi(t)\right)^2 - Igy \cdot \cos(\theta(t)) \cdot \frac{d}{dt}\phi(t) \cdot Nr + Igx \cdot \frac{d^2}{dt^2}\phi(t)$$

$$Tge(t) = (Igx + Igy - Igz) \cdot \cos(\theta(t)) \cdot \left(\frac{d}{dt}\phi(t)\right) \cdot \left(\frac{d}{dt}\theta(t)\right) + Igy \cdot \left(\frac{d}{dt}Nr(t) + \sin(\theta(t)) \cdot \frac{d^2}{dt^2}\phi(t)\right)$$

$$Tgz(t) = (Igy - Igz - Igx) \cdot \sin(\theta(t)) \cdot \left(\frac{d}{dt}\phi(t)\right) \cdot \left(\frac{d}{dt}\theta(t)\right) + Igy \cdot Nr(t) \cdot \left(\frac{d}{dt}\theta(t)\right) + Igz \cdot \cos(\theta(t)) \cdot \frac{d^2}{dt^2}\phi(t)$$

Sub-frame



Angular momentum of subframe in subframe axes:

$$Ls(t) = \begin{pmatrix} Isx\cdot\frac{d}{dt}\theta(t) \\ Isy\cdot\sin(\theta(t))\cdot\frac{d}{dt}\phi(t) \\ Isz\cdot\cos(\theta(t))\cdot\frac{d}{dt}\phi(t) \end{pmatrix}$$

$$Ts(t) = \begin{pmatrix} Tse(t) - Tgx(t) \\ Tsy(t) \\ Tsz(t) - Tgz(t) \end{pmatrix}$$

Tsz and Tsy are torques, acting on subframe, due to bearings
while Tse is torque, acting on subframe, due to linkarm.

So based on $Ts(t) = \left(\frac{d}{dt}Ls(t)\right) + \omega s(t) \times Ls(t)$

the equations of motion for subframe are as follows:

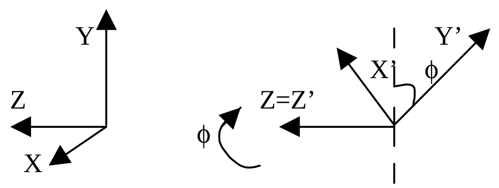
$$Tse(t) - Tgx(t) = (Isz - Isy)\cdot\sin(\theta(t))\cdot\cos(\theta(t))\cdot\left(\frac{d}{dt}\phi(t)\right)^2 + Isx\cdot\frac{d^2}{dt^2}\theta(t)$$

$$Tsy(t) = (Isx + Isy - Isz)\cdot\cos(\theta(t))\cdot\left(\frac{d}{dt}\phi(t)\right)\cdot\left(\frac{d}{dt}\theta(t)\right) + Isy\cdot\sin(\theta(t))\cdot\frac{d^2}{dt^2}\phi(t)$$

$$Tsz(t) - Tgz(t) = (Isy - Isz - Isx)\cdot\sin(\theta(t))\cdot\left(\frac{d}{dt}\phi(t)\right)\cdot\left(\frac{d}{dt}\theta(t)\right) + Isz\cdot\sin(\theta(t))\cdot\frac{d^2}{dt^2}\phi(t)$$

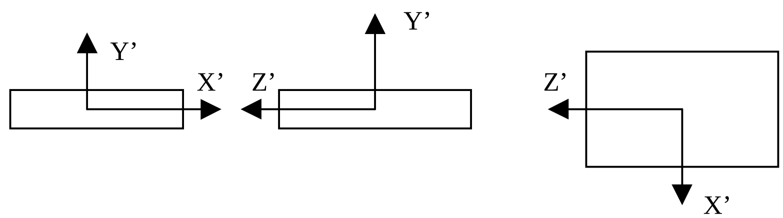
Main-frame

Choice of axes for main-frame



rotation about z axis

x', y' and z' axes are fixed with respect to mainframe as shown below,



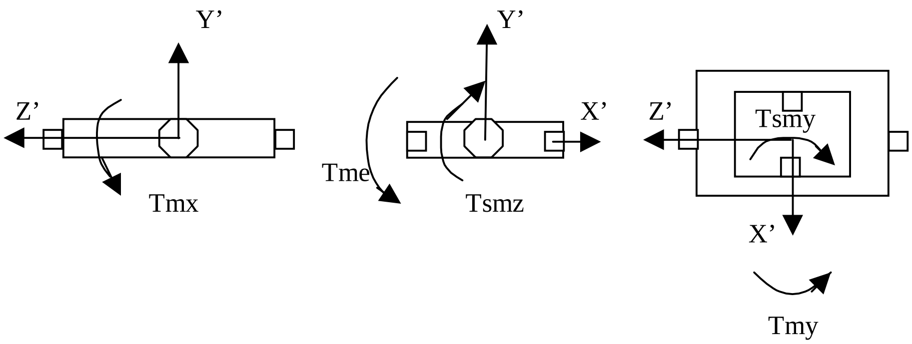
When ϕ=0, then the x',y',z' coincide with x, y, z axes.

Angular velocity of main frame in x', y', z' axes $\omega m(t) = \begin{pmatrix} 0 \\ 0 \\ \frac{d}{dt}\phi(t) \end{pmatrix}$

Angular momentum of mainframe in x', y', z' axes $Lm(t) = \begin{pmatrix} 0 \\ 0 \\ Imz \cdot \frac{d}{dt}\phi(t) \end{pmatrix}$

Transform subframe torques to mainframe, so:

$$Tsm(t) = \begin{pmatrix} Tsmx(t) \\ Tsmy(t) \\ Tsmz(t) \end{pmatrix} = \begin{pmatrix} 0 \\ Tsy\cdot\cos(\theta(t)) - Tsz\cdot\sin(\theta(t)) \\ Tsy\cdot\sin(\theta(t)) + Tsz\cdot\cos(\theta(t)) \end{pmatrix}$$



Equations of motion of mainframe:

$$Tmx(t) = 0$$

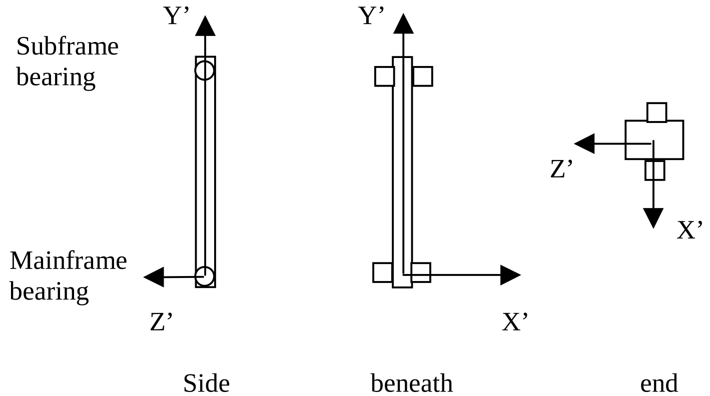
$$Tmy(t) - Tsmy(t) = 0$$

$$Tme(t) - Tsmz(t) = Imz\cdot\frac{d^2}{dt^2}\phi(t)$$

Tmx(t) and Tmy(t) are torques acting on the mainframe due to bearings, while Tme(t) is external torque acting on GVT Rotor through output shaft.

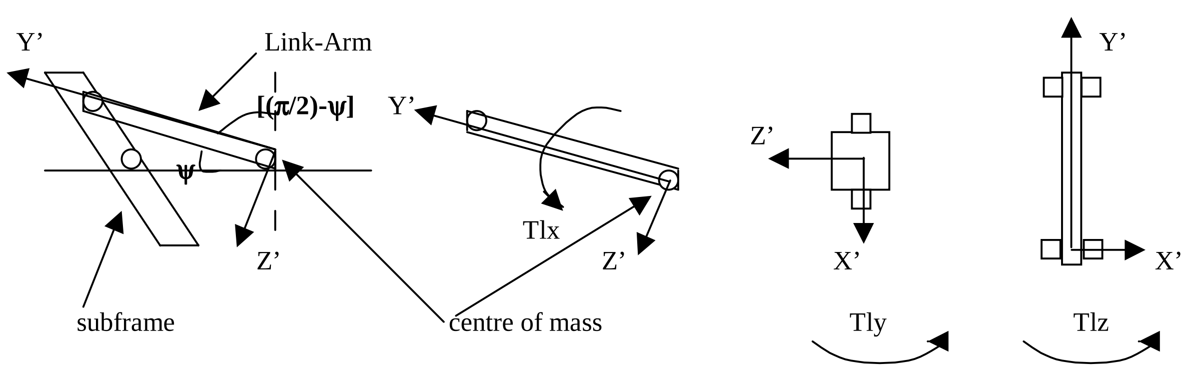
Link-Arm

x', y', z' axes fixed with respect to link-arm, as shown below.
The origin is at mainframe bearing.
The centre of mass of the link-arm is assumed to coincide with mainframe bearing.



When ψ = 90°, they coincide with x, y, z axes.

r = length of the link arm.



Tlx, Tly and Tlz are torques acting on link-arm due to bearings.
Tly and Tlz have components from mainframe bearing and subframe bearing.

Angular velocity of Link-Arm

$$\omega l(t) = \begin{bmatrix} -\left(\frac{d}{dt}\psi(t)\right) \\ \cos(\psi(t))\cdot\frac{d}{dt}\phi(t) \\ \sin(\psi(t))\cdot\frac{d}{dt}\phi(t) \end{bmatrix}$$

Anguar momentum of Link-Arm

$$Ll(t) = \begin{pmatrix} -Ilx\cdot\frac{d}{dt}\psi(t) \\ Ily\cdot\cos(\psi(t))\cdot\frac{d}{dt}\phi(t) \\ Ilz\cdot\sin(\psi(t))\cdot\frac{d}{dt}\phi(t) \end{pmatrix}$$

So, based on the equation $Tl(t) = \frac{d}{dt}Ll(t) + \omega l(t) \times Ll(t)$

the equations of motion for Link-Arm are as below

$$Tlx(t) = -Ilx\cdot\left(\frac{d^2}{dt^2}\psi(t)\right) + (Ilz - Ily)\cdot\sin(\psi(t))\cdot\cos(\psi(t))\cdot\left(\frac{d}{dt}\phi(t)\right)^2$$

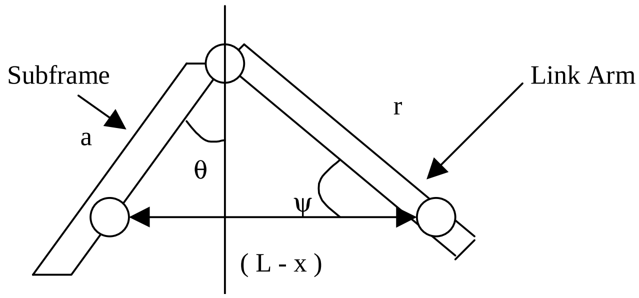
$$Tly(t) = Ily\cdot\left(\frac{d^2}{dt^2}\phi(t)\right)\cdot\cos(\psi(t)) + (Ilz - Ilx - Ily)\cdot\sin(\psi(t))\cdot\left(\frac{d}{dt}\phi(t)\right)\cdot\left(\frac{d}{dt}\psi(t)\right)$$

$$Tlz(t) = Ily\cdot\left(\frac{d^2}{dt^2}\phi(t)\right)\cdot\cos(\psi(t)) + (Ilx - Ily + Ilz)\cdot\cos(\psi(t))\cdot\left(\frac{d}{dt}\phi(t)\right)\cdot\left(\frac{d}{dt}\psi(t)\right)$$

Transform torques into mainframe axes:

$$Tlm(t) = \begin{bmatrix} Tlx(t) \\ (Tly(t)\cdot\sin(\psi(t)) - Tlz(t)\cdot\cos(\psi(t))) \\ (Tly(t)\cdot\cos(\psi(t)) + Tlz(t)\cdot\sin(\psi(t))) \end{bmatrix}$$

Geometry



$$a^2 + (L - x(t))^2 + 2\cdot a\cdot (L - x(t))\cdot \sin(\theta(t)) = r^2$$

$$\theta = \operatorname{asin}\left[\frac{\left[r^2 - a^2 - (L - x(t))^2\right]}{2\cdot a\cdot (L - x(t))}\right]$$

On differentiating,

$$\frac{d}{dt}(\theta(t)) = \left[a\cdot \sin(\theta(t)) + (L - x(t))\right]\cdot \frac{\frac{d}{dt}x(t)}{a\cdot (L - x(t))\cdot \cos(\theta(t))}$$

and differentiating again,

$$\frac{d^2}{dt^2}\theta(t) = \Psi(t) + \Omega(t)$$

$$\Psi(t) = \frac{-\left(\frac{d}{dt}x(t)\right)^2 + a\cdot (L - x(t))\cdot \sin(\theta(t))\cdot \left(\frac{d}{dt}\theta(t)\right)^2}{a\cdot (L - x(t))\cdot \cos(\theta(t))}$$

$$\Omega(t) = \frac{2\cdot a\cdot \cos(\theta(t))\cdot \left(\frac{d}{dt}x(t)\right)\cdot \left(\frac{d}{dt}\theta(t)\right) + a\cdot \sin(\theta(t))\cdot \left(\frac{d^2}{dt^2}x(t)\right) + (L - x(t))\cdot \left(\frac{d^2}{dt^2}x(t)\right)}{\left[a\cdot (L - x(t))\cdot \cos(\theta(t))\right]}$$

$$\frac{r}{\cos(\theta(t))} = \frac{a}{\sin(\psi(t))}$$

$$\psi(t) = \text{asin}\left(\frac{a \cdot \cos(\theta(t))}{r}\right)$$

On differentiating,

$$\frac{d}{dt}\psi(t) = \frac{-\left(a \cdot \sin(\theta(t)) \cdot \frac{d}{dt}\theta(t)\right)}{r \cdot \cos(\psi(t))}$$

and differentiating again

$$\frac{d^2}{dt^2}\psi(t) = \frac{\left[r \cdot \sin(\psi(t)) \cdot \left(\frac{d}{dt}\psi(t)\right)^2 - a \cdot \cos(\theta(t)) \cdot \left(\frac{d}{dt}\theta(t)\right)^2 - a \cdot \sin(\theta(t)) \cdot \frac{d}{dt}\frac{d}{dt}\theta(t)\right]}{r \cdot \cos(\psi(t))}$$

With $\theta' = \frac{d}{dt}\theta$ $\quad \psi' = \frac{d}{dt}\psi$ $\quad x' = \frac{d}{dt}x$

$$\frac{\partial}{\partial x}\theta = \frac{(L - x(t) + a\cdot\sin(\theta(t)))}{[a\cdot(L - x(t))\cdot\cos(\theta(t))]}$$

$$\frac{\partial}{\partial x}\theta' = \frac{\left[a\cdot\cos(\theta(t))\cdot\theta' + a\cdot(L - x(t))\cdot\sin(\theta(t))\cdot\theta'\cdot\frac{\partial}{\partial x}\theta + a\cdot\cos(\theta(t))\cdot\frac{\partial}{\partial x}\theta\cdot x' - x'\right]}{a\cdot(L - x(t))\cdot\cos(\theta(t))}$$

$$\frac{\partial}{\partial x'}\theta' = \frac{(a\cdot\sin(\theta(t)) + L - x(t))}{a\cdot(L - x(t))\cdot\cos(\theta(t))}$$

$$\frac{\partial}{\partial x}\psi = \frac{-(a\cdot\sin(\theta(t)))\cdot\frac{\partial}{\partial x}\theta}{(r\cdot\cos(\psi(t)))}$$

$$\frac{\partial}{\partial x}\psi' = \frac{\left(r\cdot\sin(\psi(t))\cdot\psi'\cdot\frac{\partial}{\partial x}\psi - a\cdot\cos(\theta(t))\cdot\theta'\cdot\frac{\partial}{\partial x}\theta - a\cdot\sin(\theta(t))\cdot\frac{\partial}{\partial x}\theta'\right)}{r\cdot\cos(\psi(t))}$$

$$\frac{\partial}{\partial x'}\psi' = -a\cdot\frac{\sin(\theta(t))\cdot\frac{\partial}{\partial x'}\theta'}{r\cdot\cos(\psi(t))}$$

Equation of motion for x

$$\mathrm{Fle1}(t) = Ml \cdot x'' + (Igx + Isx) \cdot \left[\frac{d}{dt} \left(\theta' \cdot \frac{\partial}{\partial x'} \theta' \right) \right] + Ilx \cdot \left[\frac{d}{dt} \left(\psi' \cdot \frac{d}{dx'} \psi' \right) \right]$$

$$\mathrm{Fle2}(t) = \left[(Igx + Isx) \cdot \theta' \cdot \frac{\partial}{\partial x'} \theta' + [(Igy + Isy) - (Igz + Isz)] \cdot (\phi')^2 \cdot \sin(\theta(t)) \cdot \cos(\theta(t)) \cdot \frac{\partial}{\partial x} \theta \right]$$

$$\mathrm{Fle3}(t) = \left[Igy \cdot Nr(t) \cdot \phi' \cdot \cos(\theta(t)) \cdot \frac{\partial}{\partial x} \theta + Ilx \cdot \psi' \cdot \frac{\partial}{\partial x} \psi' + (Ilz - Ily) \cdot (\phi')^2 \cdot \sin(\psi(t)) \cdot \cos(\psi(t)) \cdot \frac{\partial}{\partial x} \psi \right]$$

$$\mathrm{Fle}(t) = \mathrm{Fle1}(t) - \mathrm{Fle2}(t) - \mathrm{Fle3}(t)$$

Fle is the external force on the input.

Equations of motion for ϕ

$$\mathrm{Tme1}(t) = (\phi'') \cdot \left[(Igy + Isy) \cdot \sin(\theta(t))^2 + (Igz + Isz) \cdot \cos(\theta(t))^2 + Imz + Ily \cdot \cos(\psi(t))^2 + Ilz \cdot \sin(\psi(t))^2 \right]$$

$$\mathrm{Tme2}(t) = 2 \cdot (\phi') \cdot \left[(Igy + Isy) \cdot \sin(\theta(t)) \cdot \cos(\theta(t)) \cdot (\theta') - (Igz + Isz) \cdot \cos(\theta(t)) \cdot \sin(\theta(t)) \cdot (\theta') \right]$$

$$\mathrm{Tme3}(t) = 2 \cdot (\phi') \cdot \left[(Ilz - Ily) \cdot \sin(\psi(t)) \cdot \cos(\psi(t)) \cdot (\psi') \right]$$

$$\mathrm{Tme4}(t) = Igy \cdot \cos(\theta(t)) \cdot (\theta') \cdot Nr(t) + Igy \cdot \sin(\theta(t)) \cdot (Nr')$$

$$\mathrm{Tme}(t) = \mathrm{Tme1}(t) + \mathrm{Tme2}(t) + \mathrm{Tme3}(t) + \mathrm{Tme4}(t)$$

Tme is the external torque on the output shaft.

Equation of motion for Nr

$$Tge(t) = Igy\cdot\left(\phi''\cdot\sin(\theta(t)) + \phi'\cdot\cos(\theta(t))\cdot\theta' + Nr'\right)$$

Tge is the external torque on the GVT rotor.

Estimation of average power transmitted

Equation of motion for Nr

$$Tge(t) = \frac{d}{dt}\left[Igy\cdot\left(Nr(t) + \sin(\theta(t))\cdot\frac{d}{dt}\phi(t)\right)\right]$$

Assume that Tge(t) = 0, then $Igy\cdot\left(Nr(t) + \sin(\theta)\cdot\frac{d}{dt}\phi(t)\right) = \text{constant}$

Equation of motion for ϕ

$$\frac{d}{dt}(A1 + A2)(t) = Tme(t)$$

$$A1 = Igy\cdot\left(Nr(t) + \sin(\theta(t))\cdot\frac{d}{dt}\theta(t)\right)\cdot\sin(\theta(t))$$

$$A2 = \left(Igz\cdot\cos(\theta(t))^2 + Isy\cdot\sin(\theta(t))^2 + Isz\cdot\cos(\theta(t))^2 + Imz + Ily\cdot\cos(\psi(t))^2 + Ilz\cdot\sin(\psi(t))^2\right)\cdot\frac{d}{dt}\phi(t)$$

Since $\left(Nr(t) + \sin(\theta(t))\cdot\frac{d}{dt}\phi(t)\right)$ is constant,

θ and ψ are cyclic with same period (period of x),

then, assuming $\frac{d}{dt}\phi(t)$ is also constant, Tme is periodic.

Furthermore, the work done over a period is:

$$W = \int_0^T Tme(t)\cdot\left(\frac{d}{dt}\phi(t)\right)dt = 0$$

The dominant term in above is the first (A).
Tme is positive for approximately the same time that

$$\frac{d}{dt}\left[Igy\cdot\left(Nr(t) + \sin(\phi(t))\cdot\frac{d}{dt}\phi(t)\right)\cdot\sin(\theta(t))\right]$$ is positive.

In other words, when $\cos\theta \frac{d}{dt}\theta(t)$ is positive, namely $\frac{d}{dt}\theta(t)$ positive, between θmin and θmax.

Neglects the terms in A2 related to ψ(t) (Ily and Ilz beeing small)

$$WTpositive = A3 + A4$$

$$A3 = Igy\cdot\left(Nr + \sin(\theta)\cdot\frac{d}{dt}\phi(t)\right)\cdot(\sin(\theta max) - \sin(\theta min))\cdot\left(\frac{d}{dt}\phi(t)\right)$$

$$A4 = \left[Isy\cdot\left(\sin(\theta max)^2 - \sin(\theta min)^2\right) + (Igz + Isz)\cdot\left(\cos(\theta max)^2 - \cos(\theta min)^2\right)\right]\cdot\left(\frac{d}{dt}\phi(t)\right)^2$$

and with $\theta min = -\theta max$

$$WTpositive = Igy\cdot\left(Nr + \sin(\theta(t))\cdot\frac{d}{dt}\phi(t)\right)\cdot 2\cdot\sin(\theta max)\cdot\frac{d}{dt}\phi(t)$$

Average torque over one cycle (keeping only positive torque) = Tavc

$$Tavc = \frac{\left[Igy\cdot\left(Nr + \sin(\theta(t))\cdot\frac{d}{dt}\phi(t)\right)\cdot 2\cdot\sin(\theta max)\right]}{\frac{(2\pi)}{\Omega x}}$$

Average power transmitted to output = Pav

$$Pav = \left(Igy\cdot Nr0\cdot\sin(\theta max)\cdot\Omega x\cdot\frac{d}{dt}\phi(t)\right)\cdot\frac{1}{\pi}$$

where Ωx = freequency of input shaft.

With $\theta min \neq -\theta max$

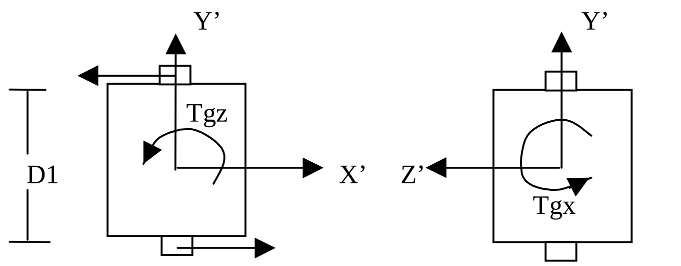
$$Pav = \frac{Igy \cdot \left[Nr0 \cdot (\sin(\theta max) - \sin(\theta min)) \cdot \Omega x \cdot \frac{d}{dt}\phi \right]}{2 \cdot \pi} + \frac{\left[(Isy - Igz - Isz) \cdot \left(\sin(\theta max)^2 - \sin(\theta min)^2 \right) \cdot \Omega x \cdot \frac{d}{dt}\phi \right]}{2 \cdot \pi}$$

APPENDIX B

GVT FORCES

Analysis of the Forces

Forces on the GVT Rotor



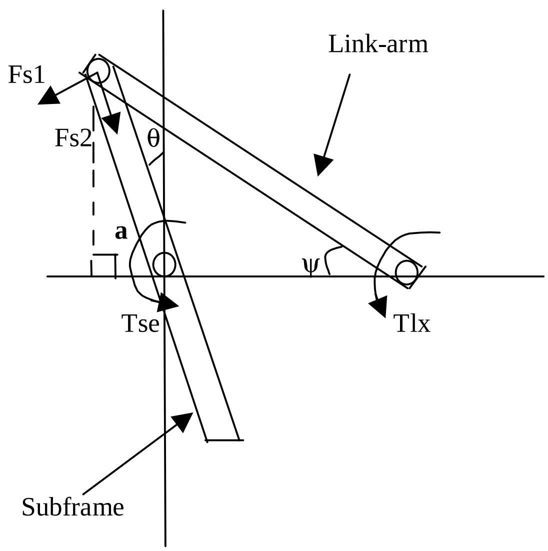
Force = ± (Tgz / D1)   Force = ± (Tgx / D1)

So, the total force of the GVT rotor is:

$$Fg(t) = \frac{\sqrt{Tgx(t)^2 + Tgz(t)^2}}{D1}$$

Forces on the subframe

Combined Forces on Subframe



$$Tse(t) = Fs1(t) \cdot a$$

$$Tlx(t) = -Fs1(t) \cdot r \cdot \sin(\theta(t) + \psi(t)) - Fs2(t) \cdot r \cdot \cos(\theta(t) + \psi(t))$$

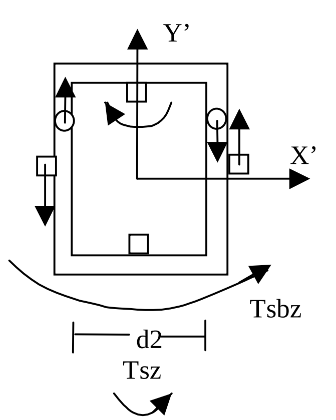
So, based on the above equations,

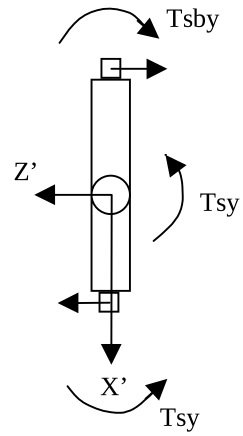
$$Fs1(t) = \frac{Tse(t)}{a}$$

$$Fs2(t) = \frac{-(Tlx(t) \cdot a + Tse(t) \cdot r \cdot \sin(\theta(t) + \psi(t)))}{a \cdot r \cdot \cos(\theta(t) + \psi(t))}$$

Differential forces on subframe

Transform of link-arm torques to subframe axes:





$$\begin{pmatrix} Tlxs(t) \\ Tlys(t) \\ Tlzs(t) \end{pmatrix} = \begin{pmatrix} Tly(t) \\ \sin(\theta(t)+\psi(t))\cdot Tly(t) - \cos(\theta(t)+\psi(t))\cdot Tlz(t) \\ \cos(\theta(t)+\psi(t))\cdot Tly(t) + \sin(\theta(t)+\psi(t))\cdot Tlz(t) \end{pmatrix}$$

Atributing totally to upper link-arm bearing, the torques on the subframe bearing are

$$Tsby(t) = Tsy(t) + (\sin(\theta(t)+\psi(t))\cdot Tly(t) - \cos(\theta(t)+\psi(t))\cdot Tlz(t))$$

$$Tsbz(t) = Tsz(t) + (\cos(\theta(t)+\psi(t))\cdot Tly(t) + \sin(\theta(t)+\psi(t))\cdot Tlz(t))$$

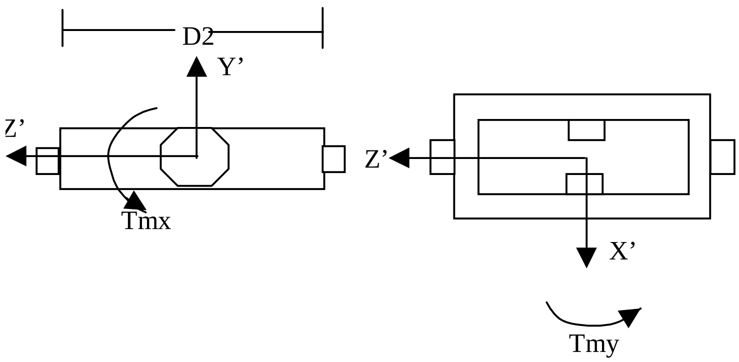
Total forces on subframe bearings:

$$Fsb(t) = \sqrt{\frac{(K(t)+\Lambda(t))}{d2}}$$

$$K(t) = (|Tsy(t)| + |\sin(\theta(t)+\psi(t))\cdot Tly(t) - \cos(\theta(t)+\psi(t))\cdot Tlz(t)|)^2$$

$$\Lambda(t) = (|Tsz(t)| + |\cos(\theta(t)+\psi(t))\cdot Tly(t) + \sin(\theta(t)+\psi(t))\cdot Tlz(t)|)^2$$

Forces of the Mainframe:



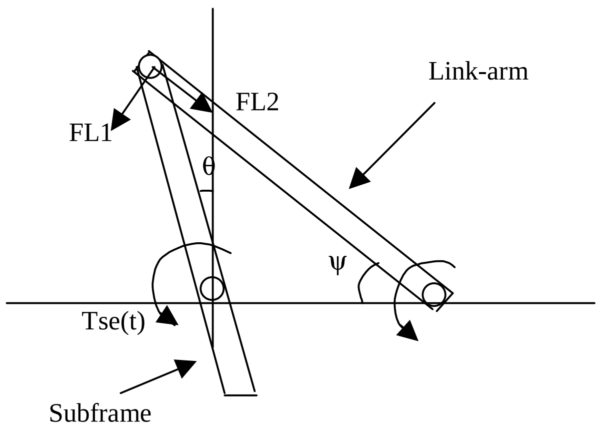
Force = $\pm$ (Tmx / D2) Force = $\pm$ (Tmy / D2)

Forces of the Mainframe:

$$Fm(t) = \frac{Tmy(t)}{D2}$$

Forces of the Link-Arm

Combined forces on top bearing of the Link-Arm:



$$\mathrm{Tlx}(t) = r \cdot \mathrm{FL1}(t)$$

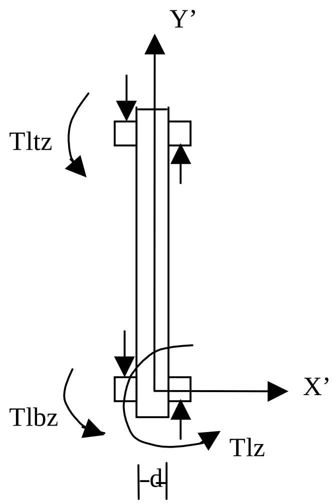
$$\mathrm{Tse}(t) = -\mathrm{FL1}(t) \cdot a \cdot \sin(\theta(t) + \psi(t)) + \mathrm{FL2}(t) \cdot a \cdot \cos(\theta(t) + \psi(t))$$

Based on the above equations:

$$\mathrm{FL1}(t) = \frac{\mathrm{Tlx}(t)}{r}$$

$$\mathrm{FL2}(t) = \frac{r \cdot \mathrm{Tse}(t) + \mathrm{Tlx}(t) \cdot a \cdot \sin(\theta(t) + \psi(t))}{a \cdot r \cdot \cos(\theta(t) + \psi(t))}$$

Differential forces on top bearing of link-arm.

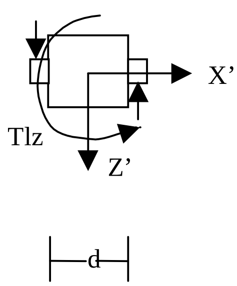


For maximum loads assume

Tlbz(t) = 0

Forces at the top bearing in Y' direction:

$FL2(t) \pm Tlz(t) / d$



For the maximum load assume

Tlby(t) = 0

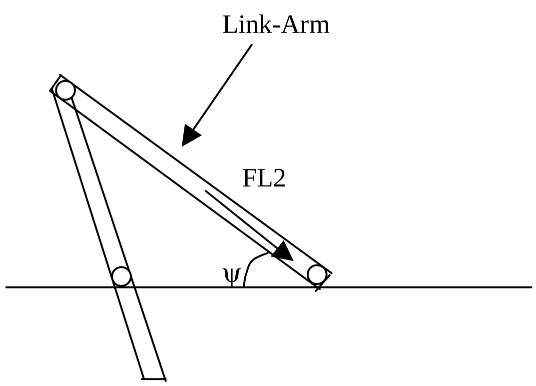
Forces at the top bearing in Z' direction:

$FL1(t) \pm Tly(t) / d$

Maximum load on top bearing is

$$Fltb(t) = \sqrt{\left(\left|FL1(t)\right| + \left|\frac{Tlz(t)}{d}\right|\right)^2 + \left(\left|FL2(t)\right| + \left|\frac{Tly(t)}{d}\right|\right)^2}$$

Combined forces on bottom bearing of the Link-Arm:



Differential forces at the bottom bearing of the link-arm.
For minimum load assume

$Tlzz(t) = 0$

Forces at the bottom bearing in Y' direction

$FL2(t) \pm Tlz(t)/d$

For maximum load assume $Tlby(t) = 0$

Forces at the bottom bearing in Z' direction

$\pm Tly(t)/d$

Maximum loads on bottom bearing

$$Flbb(t) = \sqrt{\left(\frac{Tly(t)}{d}\right)^2 + \left(\left|FL2(t)\right| + \left|\frac{Tlz(t)}{d}\right|\right)^2}$$

APPENDIX C

GVT CALCULATIONS

GVT SYSTEM CALCULATION

$\mathbf{A} := 0.12$

$\mathbf{r} := 0.4$

$\mathbf{a} := 0.2$

$\mathbf{L} := \sqrt{\mathbf{r}^2 - \mathbf{a}^2 + \mathbf{A}^2}$ $\qquad \mathbf{L} = 0.367$

$\mathbf{\Omega x} := 2.514$

$\mathbf{Ml} := 25$

$\mathbf{n} := 25$

$\mathbf{T} := 2 \cdot \frac{\boldsymbol{\pi}}{\mathbf{\Omega x}}$

$\mathbf{\Delta t} := \frac{\mathbf{T}}{\mathbf{n}}$

$\mathbf{tt} := 0, 1 .. (\mathbf{n} - 1)$

$\mathbf{t(tt)} := \mathbf{tt} \cdot \mathbf{\Delta t}$

$\mathbf{Igz} := 80$ $\qquad \mathbf{Isx} := 50$

$\mathbf{Igy} := 100$ $\qquad \mathbf{Isy} := 40$

$\mathbf{Igx} := 80$ $\qquad \mathbf{Isz} := 60$

$$\mathbf{Imz} := 120$$

$$\mathbf{Ilx} := 0.4$$

$$\mathbf{Ily} := 0.1$$

$$\mathbf{Ilz} := 0.4$$

$$\mathbf{D1} := \frac{1}{2} \qquad \mathbf{d} := 0.1$$

$$\mathbf{D2} := \frac{3}{2} \qquad \mathbf{d2} := 0.8$$

$$\mathbf{Nr0} := 500$$

Gyro size and mass

$$\mathbf{L_g} := 1$$

$$\rho_s := 7860$$

$$\mathbf{R_g} := \left(\frac{2 \cdot \mathbf{Igy}}{\pi \cdot \rho_s \cdot \mathbf{L_g}}\right)^{0.25}$$

$$\mathbf{R_g} = 0.3$$

$$\mathbf{M_g} := \rho_s \cdot \pi \cdot \mathbf{R_g}^2 \cdot \mathbf{L_g}$$

$$\mathbf{M_g} = 2.222 \times 10^3$$

GLOSSARY

a = lengh of subframe, which is between the bearing of gvt rotor to subframe and the bearing of link-arm to subframe.

d = width of the link-arm
d2 = width of the subframe
D1 = lengh of the gvt rotor
D2 = width of mainframe

Fg = total force on the gvt rotor
Fm = total force on the mainframe
Fle = external force on the input shaft
Flbb = magnitude of total force on the bottom bearing of the link-arm
Fltb = magnitude of total force on the top bearing of the link-arm
Fsb = magnitude of total force on subframe bearings

Igx = moment of inertia of gvt rotor in x axes
Igy = moment of inertia of gvt rotor in y axes
Igz = moment of inertia of gvt rotor in z axes

Isx = moment of inertia of subframe in x axes
Isy = moment of inertia of subframe in y axes
Isz = moment of inertia of subframe in z axes

Imz = moment of inertia of mainframe in z axes

Ilx = moment of inertia of link-arm in x axes
Ily = moment of inertia of link-arm in y axes
Ilz = moment of inertia of link-arm in z axes

Lg = angular momentum of gyroscope
Ls = angular momentum of subframe
Lm = angular momentum of mainframe
Ll = angular momentum of link-arm

Nr = gvt rotor speed

$$\mathbf{Nrd(t) = \frac{d}{dt}Nr(t)}$$

Pav = average power transmitted to output

r = length of the link-arm

Tgx = internal torque on gvt rotor about x axes
Tge = external torque on gvt rotor about y axes
Tgz = internal torque on gvt rotor about z axes

Tse = internal torque on subframe about x axes
Tsy = internal torque on subframe about y axes
Tsz = internal torque on subframe about z axes

Tme = external torque on the output shaft about z axes
Tmy = internal torque on mainframe about y axes

Tlx = internal torque on link-arm about x axes
Tly = internal torque on link-arm about y axes
Tlz = internal torque on link-arm about z axes

$\mathbf{x}$ = position of input shaft

$$\mathbf{x(t) := A \cdot sin(\Omega x \cdot t)}$$

$$\mathbf{xd(t) := \frac{d}{dt} x(t)}$$

$$\mathbf{xdd(t) := \frac{d^2}{dt^2} x(t)}$$

θ = angular displacement of subframe relating to mainframe
ϕ = angular displacement of input shaft
ψ = angular displacement of link-arm relating to maiframe

$$\theta(\mathbf{t}) = \mathbf{asin}\left[\frac{\left[\mathbf{r}^2 - \mathbf{a}^2 - (\mathbf{L} - \mathbf{x(t)})^2\right]}{2 \cdot \mathbf{a} \cdot (\mathbf{L} - \mathbf{x(t)})}\right]$$

$$\theta\mathbf{d(t)} = \left[\mathbf{a} \cdot \mathbf{sin}(\theta(\mathbf{t})) + (\mathbf{L} - \mathbf{x(t)})\right] \cdot \frac{(\mathbf{xd(t)})}{\mathbf{a} \cdot (\mathbf{L} - \mathbf{x(t)}) \cdot \mathbf{cos}(\theta(\mathbf{t}))}$$

$$\psi(\mathbf{t}) = \mathbf{asin}\left(\mathbf{a} \cdot \frac{\mathbf{cos}(\theta(\mathbf{t}))}{\mathbf{r}}\right)$$

$$\psi\mathbf{d(t)} = \frac{\mathbf{d}}{\mathbf{dt}} \psi(\mathbf{t})$$

$$\psi\mathbf{dd(t)} = \frac{\mathbf{d}^2}{\mathbf{dt}^2} \psi(\mathbf{t})$$

$$\theta' = \frac{d}{dt}\theta$$

$$x' = \frac{d}{dt}x$$

$$\delta\theta\delta x(t) = \frac{\partial}{\partial x}\theta$$

$$\delta\theta d\delta x(t) = \frac{\partial}{\partial x}\theta'$$

$$\delta\theta d\delta xd(t) = \frac{\partial}{\partial x'}\theta'$$

$$\delta\psi\delta x(t) = \frac{\partial}{\partial x}\psi$$

$$\delta\psi d\delta x(t) = \frac{\partial}{\partial x}\psi'$$

$$\delta\psi d\delta xd(t) = \frac{\partial}{\partial x'}\psi'$$

$$\phi d(t) := 31.4$$

$$\phi dd(t) = \frac{d^2}{dt^2}\phi(t) \qquad \phi dd(t) := \frac{d}{dt}\phi d(t)$$

Ωx = freequency of input shaft

Input - Output Equations

$$\mathbf{u}(\mathbf{t}) := \frac{\left[\mathbf{r}^2 - \mathbf{a}^2 - (\mathbf{L} - \mathbf{x}(\mathbf{t}))^2\right]}{2\cdot\mathbf{a}\cdot(\mathbf{L} - \mathbf{x}(\mathbf{t}))}$$

$$\theta(\mathbf{t}) := \mathbf{asin}(\mathbf{u}(\mathbf{t}))$$

$$\theta d(t) := \left[a\cdot\sin(\theta(t)) + (L - x(t))\right]\cdot\frac{(xd(t))}{a\cdot(L - x(t))\cdot\cos(\theta(t))}$$

$$\theta dd1(t) := \frac{\left[2\cdot a\cdot\cos(\theta(t))\cdot(xd(t))\cdot(\theta d(t)) + a\cdot\sin(\theta(t))\cdot(xdd(t)) + (L - x(t))\cdot(xdd(t))\right]}{a\cdot(L - x(t))\cdot\cos(\theta(t))}$$

$$\theta dd(t) := \frac{\left[-(xd(t))^2 + a\cdot(L - x(t))\cdot\sin(\theta(t))\cdot(\theta d(t))^2\right]}{a\cdot(L - x(t))\cdot\cos(\theta(t))} + \theta dd1(t)$$

$$\psi(t) := \operatorname{asin}\left(a\cdot\frac{\cos(\theta(t))}{r}\right)$$

$$\psi d(t) := \frac{-\left[a\cdot\sin(\theta(t))\cdot(\theta d(t))\right]}{r\cdot\cos(\psi(t))}$$

$$\psi dd(t) := \frac{\left[r\cdot\sin(\psi(t))\cdot(\psi d(t))^2 - a\cdot\cos(\theta(t))\cdot(\theta d(t))^2 - a\cdot\sin(\theta(t))\cdot(\theta dd(t))\right]}{r\cdot\cos(\psi(t))}$$

External Torques & Forces (Tge, Tme and Fle)

GVT Rotor:

$$\lambda := 0$$

$$\mathbf{Nr}(\mathbf{t}) := \mathbf{Nr0} - \lambda \cdot \phi\mathbf{d}(\mathbf{t}) \cdot \mathbf{sin}(\theta(\mathbf{t}))$$

There are two cases for Nr(t) :

At the first case λ=1, therefore $\mathbf{Nr}(\mathbf{t}) = \mathbf{Nr0} - \phi\mathbf{d}(\mathbf{t}) \cdot \mathbf{sin}(\theta(\mathbf{t}))$ corresponds to Tge=0

At the second case λ=0, so, $\mathbf{Nr}(\mathbf{t}) = \mathbf{Nr0}$

[redacted]

$$\mathbf{Nrd}(\mathbf{t}) := \frac{\mathbf{d}}{\mathbf{dt}}\mathbf{Nr}(\mathbf{t})$$

$$\mathbf{Tge}(\mathbf{t}) := \mathbf{Igy} \cdot \left(\phi\mathbf{dd}(\mathbf{t}) \cdot \mathbf{sin}(\theta(\mathbf{t})) + \phi\mathbf{d}(\mathbf{t}) \cdot \mathbf{cos}(\theta(\mathbf{t})) \cdot \theta\mathbf{d}(\mathbf{t}) + \mathbf{Nrd}(\mathbf{t})\right)$$

Output shaft:

$$\mathbf{Tme1}(\mathbf{t}) := (\phi\mathbf{dd}(\mathbf{t}))\cdot\left[(\mathbf{Igy}+\mathbf{Isy})\cdot\mathbf{sin}(\theta(\mathbf{t}))^2+(\mathbf{Igz}+\mathbf{Isz})\cdot\mathbf{cos}(\theta(\mathbf{t}))^2+\mathbf{Imz}+\mathbf{Ily}\cdot\mathbf{cos}(\psi(\mathbf{t}))^2+\mathbf{Ilz}\cdot\mathbf{sin}(\psi(\mathbf{t}))^2\right]$$

$$\mathbf{Tme2}(\mathbf{t}) := 2\cdot(\phi\mathbf{d}(\mathbf{t}))\cdot\left[(\mathbf{Igy}+\mathbf{Isy})\cdot\mathbf{sin}(\theta(\mathbf{t}))\cdot\mathbf{cos}(\theta(\mathbf{t}))\cdot(\theta\mathbf{d}(\mathbf{t}))-(\mathbf{Igz}+\mathbf{Isz})\cdot\mathbf{cos}(\theta(\mathbf{t}))\cdot\mathbf{sin}(\theta(\mathbf{t}))\cdot(\theta\mathbf{d}(\mathbf{t}))\right]$$

$$\mathbf{Tme3}(\mathbf{t}) := 2\cdot(\phi\mathbf{d}(\mathbf{t}))\cdot\left[(\mathbf{Ilz}-\mathbf{Ily})\cdot\mathbf{sin}(\psi(\mathbf{t}))\cdot\mathbf{cos}(\psi(\mathbf{t}))\cdot(\psi\mathbf{d}(\mathbf{t}))\right]$$

$$\mathbf{Tme4}(\mathbf{t}) := \mathbf{Igy}\cdot\mathbf{cos}(\theta(\mathbf{t}))\cdot(\theta\mathbf{d}(\mathbf{t}))\cdot\mathbf{Nr}(\mathbf{t})+\mathbf{Igy}\cdot\mathbf{sin}(\theta(\mathbf{t}))\cdot(\mathbf{Nrd}(\mathbf{t}))$$

$$\mathbf{Tme}(\mathbf{t}) := \mathbf{Tme1}(\mathbf{t})+\mathbf{Tme2}(\mathbf{t})+\mathbf{Tme3}(\mathbf{t})+\mathbf{Tme4}(\mathbf{t})$$

Input shaft:

$$\delta\theta\delta x(t) := \frac{(L - x(t) + a\cdot\sin(\theta(t)))}{[a\cdot(L - x(t))\cdot\cos(\theta(t))]}$$

$$\delta\theta d\delta x(t) := \frac{[a\cdot\cos(\theta(t))\cdot\theta d(t) + a\cdot(L - x(t))\cdot\sin(\theta(t))\cdot\theta d(t)\cdot\delta\theta\delta x(t) + a\cdot\cos(\theta(t))\cdot\delta\theta\delta x(t)\cdot xd(t) - xd(t)]}{a\cdot(L - x(t))\cdot\cos(\theta(t))}$$

$$\delta\theta d\delta xd(t) := \frac{(a\cdot\sin(\theta(t)) + L - x(t))}{a\cdot(L - x(t))\cdot\cos(\theta(t))}$$

$$\delta\psi\delta x(t) := \frac{-(a\cdot\sin(\theta(t)))\cdot\delta\theta\delta x(t)}{(r\cdot\cos(\psi(t)))}$$

$$\delta\psi d\delta x(t) := \frac{(r\cdot\sin(\psi(t))\cdot\psi d(t)\cdot\delta\psi\delta x(t) - a\cdot\cos(\theta(t))\cdot\theta d(t)\cdot\delta\theta\delta x(t) - a\cdot\sin(\theta(t))\cdot\delta\theta d\delta x(t))}{r\cdot\cos(\psi(t))}$$

$$\delta\psi d\delta xd(t) := -a\cdot\frac{\sin(\theta(t))\cdot\delta\theta d\delta xd(t)}{r\cdot\cos(\psi(t))}$$

$$\mathbf{A(t)} := \frac{d}{dt}\,\delta\theta d\delta xd(t)$$

$$\mathbf{A(t)} := \frac{\left[a\cdot xd(t)\cdot\cos(\theta(t))\cdot\delta\theta d\delta xd(t) + a\cdot(L - x(t))\cdot\sin(\theta(t))\cdot\theta d(t)\cdot\delta\theta d\delta xd(t) + a\cdot\cos(\theta(t))\cdot\theta d(t) - xd(t)\right]}{a\cdot(L - x(t))\cdot\cos(\theta(t))}$$

$$\mathbf{C(t)} := \frac{d}{dt}\left(\theta d(t)\cdot\delta\theta d\delta xd(t)\right)$$

$$\mathbf{C(t)} := \theta dd(t)\cdot\delta\theta d\delta xd(t) + \theta d(t)\cdot A(t)$$

$$\mathbf{D(t)} := \frac{d}{dt}\,\delta\psi d\delta xd(t)$$

$$\mathbf{D(t)} := -\left(a\cdot\sin(\theta(t))\cdot\sin(\psi(t))\cdot\frac{\psi d(t)}{r\cdot\cos(\psi(t))^{2}} + a\cdot\cos(\theta(t))\cdot\frac{\theta d(t)}{r\cdot\cos(\psi(t))}\right)\cdot\delta\theta d\delta xd(t) - a\cdot\frac{\sin(\theta(t))\cdot\frac{d}{dt}\delta\theta d\delta xd(t)}{r\cdot\cos(\psi(t))}$$

$$\mathbf{E(t)} := \frac{d}{dt}\left(\psi d(t)\cdot\delta\psi d\delta xd(t)\right)$$

$$\mathbf{E(t)} := \psi dd(t)\cdot\delta\psi d\delta xd(t) + \psi d(t)\cdot D(t)$$

$$\mathbf{Fle1}(\mathbf{t}) := \mathbf{Ml}\cdot\mathbf{xdd}(\mathbf{t}) + (\mathbf{Igx} + \mathbf{Isx})\cdot\mathbf{C}(\mathbf{t}) + \mathbf{Ilx}\cdot\mathbf{E}(\mathbf{t})$$

$$\mathbf{Fle2}(\mathbf{t}) := \left[(\mathbf{Igx} + \mathbf{Isx})\cdot\boldsymbol{\theta}\mathbf{d}(\mathbf{t})\cdot\boldsymbol{\delta\theta}\mathbf{d}\boldsymbol{\delta}\mathbf{x}(\mathbf{t}) + [(\mathbf{Igy} + \mathbf{Isy}) - (\mathbf{Igz} + \mathbf{Isz})]\cdot\boldsymbol{\phi}\mathbf{d}(\mathbf{t})^2\cdot\mathbf{sin}\big(\boldsymbol{\theta}(\mathbf{t})\big)\cdot\mathbf{cos}\big(\boldsymbol{\theta}(\mathbf{t})\big)\cdot\boldsymbol{\delta\theta\delta}\mathbf{x}(\mathbf{t}) \right]$$

$$\mathbf{Fle3}(\mathbf{t}) := \left[\mathbf{Igy}\cdot\mathbf{Nr}(\mathbf{t})\cdot\boldsymbol{\phi}\mathbf{d}(\mathbf{t})\cdot\mathbf{cos}\big(\boldsymbol{\theta}(\mathbf{t})\big)\cdot\boldsymbol{\delta\theta\delta}\mathbf{x}(\mathbf{t}) + \mathbf{Ilx}\cdot\boldsymbol{\psi}\mathbf{d}(\mathbf{t})\cdot\boldsymbol{\delta\psi}\mathbf{d}\boldsymbol{\delta}\mathbf{x}(\mathbf{t}) + (\mathbf{Ilz} - \mathbf{Ily})\cdot\boldsymbol{\phi}\mathbf{d}(\mathbf{t})^2\cdot\mathbf{sin}\big(\boldsymbol{\psi}(\mathbf{t})\big)\cdot\mathbf{cos}\big(\boldsymbol{\psi}(\mathbf{t})\big)\cdot\boldsymbol{\delta\psi\delta}\mathbf{x}(\mathbf{t}) \right]$$

$$\mathbf{Fle}(\mathbf{t}) := \mathbf{Fle1}(\mathbf{t}) - \mathbf{Fle2}(\mathbf{t}) - \mathbf{Fle3}(\mathbf{t})$$

INTERNAL TORQUES

GVT ROTOR :

$$\mathbf{Tgx(t)} := (\mathbf{Igz} - \mathbf{Igy})\cdot\left[\mathbf{sin}(\theta(\mathbf{t}))\cdot\mathbf{cos}(\theta(\mathbf{t}))\cdot(\phi\mathbf{d}(\mathbf{t}))^2\right] - (\mathbf{Igy}\cdot\mathbf{cos}(\theta(\mathbf{t}))\cdot\phi\mathbf{d}(\mathbf{t})\cdot\mathbf{Nr}(\mathbf{t}) + \mathbf{Igx}\cdot\theta\mathbf{dd}(\mathbf{t}))$$

$$\mathbf{Tge(t)} := (\mathbf{Igx} + \mathbf{Igy} - \mathbf{Igz})\cdot\mathbf{cos}(\theta(\mathbf{t}))\cdot(\phi\mathbf{d}(\mathbf{t}))\cdot(\theta\mathbf{d}(\mathbf{t})) + \mathbf{Igy}\cdot(\mathbf{Nrd}(\mathbf{t}) + \mathbf{sin}(\theta(\mathbf{t}))\cdot\phi\mathbf{dd}(\mathbf{t}))$$

$$\mathbf{Tgz(t)} := (\mathbf{Igy} - \mathbf{Igz} - \mathbf{Igx})\cdot\mathbf{sin}(\theta(\mathbf{t}))\cdot(\phi\mathbf{d}(\mathbf{t}))\cdot(\theta\mathbf{d}(\mathbf{t})) + \mathbf{Igy}\cdot\mathbf{Nr}(\mathbf{t})\cdot(\theta\mathbf{d}(\mathbf{t})) + \mathbf{Igz}\cdot\mathbf{cos}(\theta(\mathbf{t}))\cdot\phi\mathbf{dd}(\mathbf{t})$$

SUBFRAME :

$$\mathbf{Tse(t)} := (\mathbf{Isz} - \mathbf{Isy})\cdot\mathbf{sin}(\theta(\mathbf{t}))\cdot\mathbf{cos}(\theta(\mathbf{t}))\cdot(\phi\mathbf{d}(\mathbf{t}))^2 + \mathbf{Isx}\cdot\theta\mathbf{dd}(\mathbf{t}) + \mathbf{Tgx}(\mathbf{t})$$

$$\mathbf{Tsy(t)} := (\mathbf{Isx} + \mathbf{Isy} - \mathbf{Isz})\cdot\mathbf{cos}(\theta(\mathbf{t}))\cdot(\phi\mathbf{d}(\mathbf{t}))\cdot(\theta\mathbf{d}(\mathbf{t})) + \mathbf{Isy}\cdot\mathbf{sin}(\theta(\mathbf{t}))\cdot\phi\mathbf{dd}(\mathbf{t})$$

$$\mathbf{Tsz(t)} := \mathbf{Tgz}(\mathbf{t}) + \left[(\mathbf{Isy} - \mathbf{Isz} - \mathbf{Isx})\cdot\mathbf{sin}(\theta(\mathbf{t}))\cdot(\phi\mathbf{d}(\mathbf{t}))\cdot(\theta\mathbf{d}(\mathbf{t})) + \mathbf{Isz}\cdot\mathbf{sin}(\theta(\mathbf{t}))\cdot\phi\mathbf{dd}(\mathbf{t})\right]$$

MAINFRAME :

$$\mathbf{Tsm(t)} := \begin{pmatrix} 0 \\ \mathbf{Tsy(t)\cdot cos(\theta(t)) - Tsz(t)\cdot sin(\theta(t))} \\ \mathbf{Tsy(t)\cdot sin(\theta(t)) + Tsz(t)\cdot cos(\theta(t))} \end{pmatrix}$$

$$\mathbf{Tsmy(t) := Tsy(t)\cdot cos(\theta(t)) - Tsz(t)\cdot sin(\theta(t))}$$

$$\mathbf{Tmy(t) := Tsmy(t)}$$

$$\mathbf{Tsmz(t) := Tsy(t)\cdot sin(\theta(t)) + Tsz(t)\cdot cos(\theta(t))}$$

$$\mathbf{Tme(t) := Tsmz(t) + Imz\cdot \phi dd(t)}$$

LINK ARM :

$$\mathbf{Tlx(t) := -(\psi dd(t))\cdot Ilx + (Ilz - Ily)\cdot sin(\psi(t))\cdot cos(\psi(t))\cdot (\phi d(t))^2}$$

$$\mathbf{Tly(t) := (\phi dd(t))\cdot cos(\psi(t))\cdot Ily + (Ilz - Ilx - Ily)\cdot sin(\psi(t))\cdot (\phi d(t))\cdot (\psi d(t))}$$

$$\mathbf{Tlz(t) := (\phi dd(t))\cdot cos(\psi(t))\cdot Ily + (Ilx - Ily + Ilz)\cdot cos(\psi(t))\cdot (\phi d(t))\cdot (\psi d(t))}$$

Average Power Transmitted To Output :

$$\mathbf{m} := 0$$

$$\mathbf{v}(\mathbf{tt}, \mathbf{m}) := \mathbf{u}(\mathbf{tt} \cdot \Delta\mathbf{t})$$

$$\omega := \mathbf{matrix}(\mathbf{n}, 1, \mathbf{v})$$

$$\phi\mathbf{d0} := \phi\mathbf{d}(0)$$

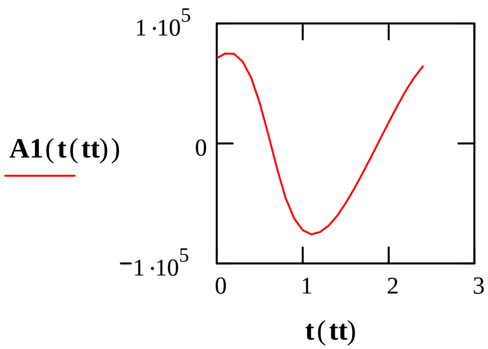
For estimate of power to be accured require $|A2(t)| << |A1(t)|$

$$\mathbf{A1}(\mathbf{t}) := \frac{d}{dt}\left[\mathbf{Igy} \cdot \left(\mathbf{Nr}(\mathbf{t}) + \sin(\theta(\mathbf{t})) \cdot \frac{d}{dt}\theta(\mathbf{t})\right) \cdot \sin(\theta(\mathbf{t}))\right]$$

$$\mathbf{A2}(\mathbf{t}) := \frac{d}{dt}\left[\left(\mathbf{Igz} \cdot \cos(\theta(\mathbf{t}))^2 + \mathbf{Isy} \cdot \sin(\theta(\mathbf{t}))^2 + \mathbf{Isz} \cdot \cos(\theta(\mathbf{t}))^2 + \mathbf{Imz} + \mathbf{Ily} \cdot \cos(\psi(\mathbf{t}))^2 + \mathbf{Ilz} \cdot \sin(\psi(\mathbf{t}))^2\right) \cdot \phi\mathbf{d}(\mathbf{t})\right]$$

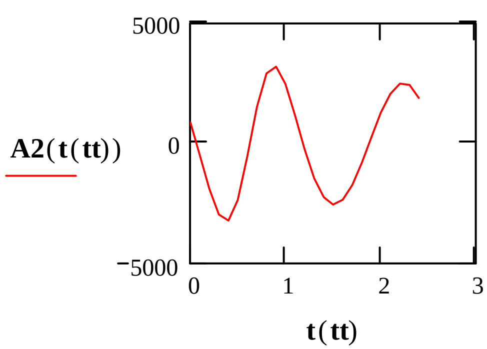
$\mathbf{A1(t(tt))} =$

$7.134\cdot10^{4}$
$7.52\cdot10^{4}$
$7.472\cdot10^{4}$
$6.838\cdot10^{4}$
$5.472\cdot10^{4}$
$3.348\cdot10^{4}$
$6.785\cdot10^{3}$
$-2.097\cdot10^{4}$
$-4.501\cdot10^{4}$
$-6.231\cdot10^{4}$
$-7.216\cdot10^{4}$
$-7.548\cdot10^{4}$
$-7.375\cdot10^{4}$
$-6.83\cdot10^{4}$
$-6.007\cdot10^{4}$
$-4.972\cdot10^{4}$



$\mathbf{A2(t(tt))} =$

881.034
-462.287
$-1.872\cdot10^{3}$
$-2.947\cdot10^{3}$
$-3.213\cdot10^{3}$
$-2.352\cdot10^{3}$
-528.707
$1.526\cdot10^{3}$
$2.927\cdot10^{3}$
$3.208\cdot10^{3}$
$2.481\cdot10^{3}$
$1.181\cdot10^{3}$
-237.177
$-1.441\cdot10^{3}$
$-2.237\cdot10^{3}$
$-2.538\cdot10^{3}$



$$\mathbf{Pav} := \frac{\mathbf{Igy}\cdot\left[\mathbf{Nr0}\cdot(\mathbf{max}(\omega)-\mathbf{min}(\omega))\cdot\mathbf{\Omega x}\cdot\mathbf{\phi d0}\right]}{2\cdot\pi} + \frac{\left[(\mathbf{Isy}-\mathbf{Igz}-\mathbf{Isz})\cdot\left(\mathbf{max}(\omega)^2-\mathbf{min}(\omega)^2\right)\cdot\mathbf{\Omega x}\cdot\mathbf{\phi d0}\right]}{2\cdot\pi}$$

Results :

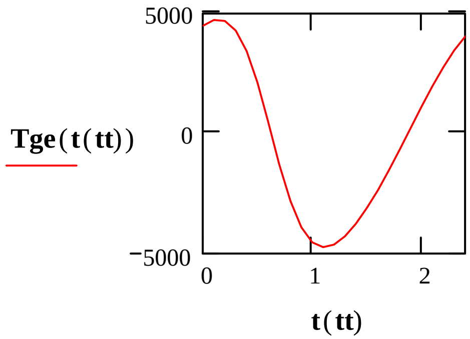
Average Power Transmitted To Output :

$$\mathbf{Pav} = 7.522 \times 10^{5}$$

External Torques & Forces :

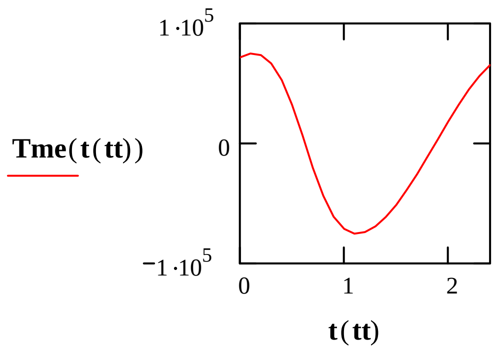
$\mathbf{Tge}(\mathbf{t}(\mathbf{tt})) =$

$4.483 \cdot 10^{3}$
$4.721 \cdot 10^{3}$
$4.687 \cdot 10^{3}$
$4.286 \cdot 10^{3}$
$3.433 \cdot 10^{3}$
$2.109 \cdot 10^{3}$
441.553
$-1.304 \cdot 10^{3}$
$-2.826 \cdot 10^{3}$
$-3.92 \cdot 10^{3}$
$-4.539 \cdot 10^{3}$
$-4.743 \cdot 10^{3}$
$-4.631 \cdot 10^{3}$
$-4.285 \cdot 10^{3}$
$-3.768 \cdot 10^{3}$
$-3.119 \cdot 10^{3}$



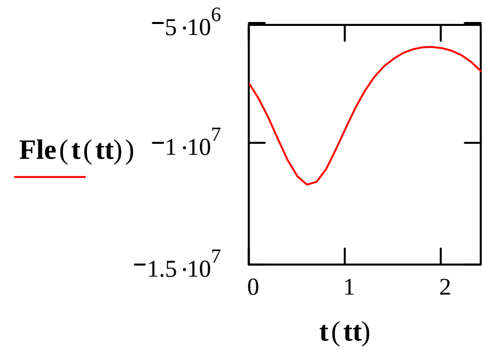
$Tme(t(tt)) =$

$7.182 \cdot 10^4$
$7.494 \cdot 10^4$
$7.37 \cdot 10^4$
$6.678 \cdot 10^4$
$5.305 \cdot 10^4$
$3.241 \cdot 10^4$
$6.767 \cdot 10^3$
$-2.001 \cdot 10^4$
$-4.353 \cdot 10^4$
$-6.081 \cdot 10^4$
$-7.103 \cdot 10^4$
$-7.494 \cdot 10^4$
$-7.385 \cdot 10^4$
$-6.896 \cdot 10^4$
$-6.111 \cdot 10^4$
$-5.093 \cdot 10^4$



$Fle(t(tt)) =$

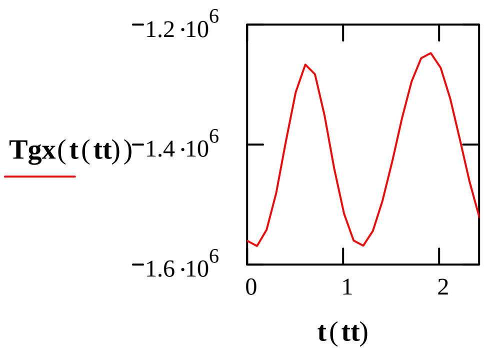
$-7.429 \cdot 10^6$
$-8.078 \cdot 10^6$
$-8.865 \cdot 10^6$
$-9.747 \cdot 10^6$
$-1.062 \cdot 10^7$
$-1.132 \cdot 10^7$
$-1.166 \cdot 10^7$
$-1.155 \cdot 10^7$
$-1.1 \cdot 10^7$
$-1.019 \cdot 10^7$
$-9.299 \cdot 10^6$
$-8.455 \cdot 10^6$
$-7.735 \cdot 10^6$
$-7.158 \cdot 10^6$
$-6.715 \cdot 10^6$
$-6.389 \cdot 10^6$



GVT Rotor:

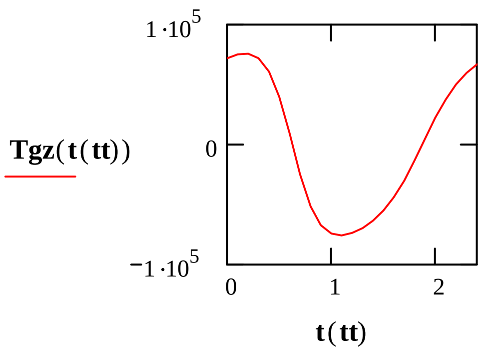
$Tgx(t(tt)) =$

$-1.561\cdot10^{6}$
$-1.569\cdot10^{6}$
$-1.542\cdot10^{6}$
$-1.481\cdot10^{6}$
$-1.396\cdot10^{6}$
$-1.313\cdot10^{6}$
$-1.267\cdot10^{6}$
$-1.283\cdot10^{6}$
$-1.352\cdot10^{6}$
$-1.44\cdot10^{6}$
$-1.515\cdot10^{6}$
$-1.56\cdot10^{6}$
$-1.569\cdot10^{6}$
$-1.544\cdot10^{6}$
$-1.494\cdot10^{6}$
$-1.427\cdot10^{6}$



$Tgz(t(tt)) =$

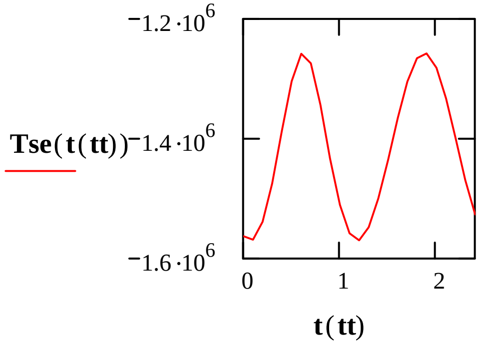
$7.199\cdot10^{4}$
$7.512\cdot10^{4}$
$7.559\cdot10^{4}$
$7.174\cdot10^{4}$
$6.075\cdot10^{4}$
$3.96\cdot10^{4}$
$8.577\cdot10^{3}$
$-2.503\cdot10^{4}$
$-5.156\cdot10^{4}$
$-6.734\cdot10^{4}$
$-7.435\cdot10^{4}$
$-7.577\cdot10^{4}$
$-7.383\cdot10^{4}$
$-6.966\cdot10^{4}$
$-6.353\cdot10^{4}$
$-5.519\cdot10^{4}$



Subframe :

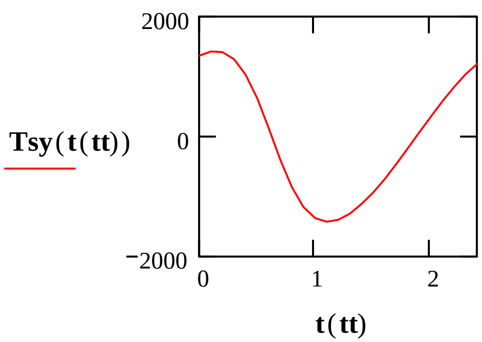
$\mathbf{Tse}(\mathbf{t}(\mathbf{tt})) =$

$-1.562\cdot10^{6}$
$-1.568\cdot10^{6}$
$-1.538\cdot10^{6}$
$-1.474\cdot10^{6}$
$-1.388\cdot10^{6}$
$-1.304\cdot10^{6}$
$-1.258\cdot10^{6}$
$-1.274\cdot10^{6}$
$-1.343\cdot10^{6}$
$-1.433\cdot10^{6}$
$-1.51\cdot10^{6}$
$-1.558\cdot10^{6}$
$-1.57\cdot10^{6}$
$-1.548\cdot10^{6}$
$-1.499\cdot10^{6}$
$-1.434\cdot10^{6}$



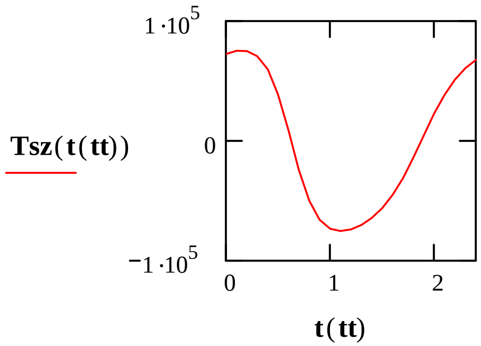
$$\mathbf{Tsy}(\mathbf{t}(\mathbf{tt})) =$$

$1.345\cdot10^{3}$
$1.416\cdot10^{3}$
$1.406\cdot10^{3}$
$1.286\cdot10^{3}$
$1.03\cdot10^{3}$
632.828
132.466
-391.342
-847.72
$-1.176\cdot10^{3}$
$-1.362\cdot10^{3}$
$-1.423\cdot10^{3}$
$-1.389\cdot10^{3}$
$-1.286\cdot10^{3}$
$-1.13\cdot10^{3}$
-935.711



$$\mathbf{Tsz}(\mathbf{t}(\mathbf{tt})) =$$

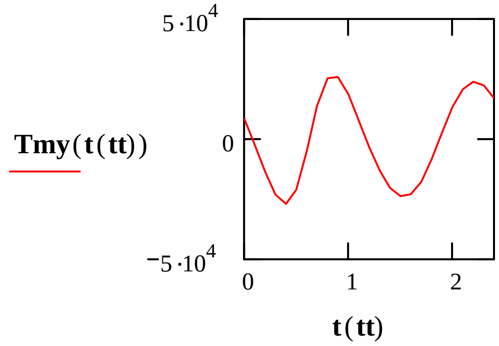
$7.23\cdot10^{4}$
$7.496\cdot10^{4}$
$7.492\cdot10^{4}$
$7.064\cdot10^{4}$
$5.948\cdot10^{4}$
$3.861\cdot10^{4}$
$8.346\cdot10^{3}$
$-2.437\cdot10^{4}$
$-5.037\cdot10^{4}$
$-6.611\cdot10^{4}$
$-7.345\cdot10^{4}$
$-7.535\cdot10^{4}$
$-7.391\cdot10^{4}$
$-7.017\cdot10^{4}$
$-6.435\cdot10^{4}$
$-5.617\cdot10^{4}$



Mainframe :

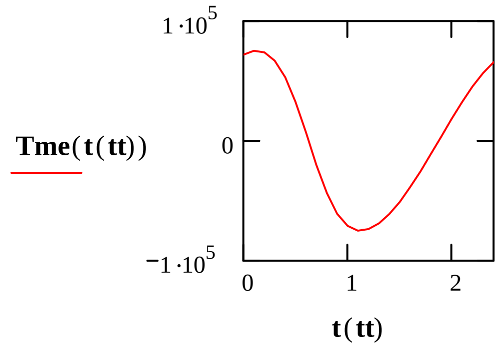
$\mathbf{Tmy(t(tt))} =$

$8.438 \cdot 10^{3}$
$-2.253 \cdot 10^{3}$
$-1.357 \cdot 10^{4}$
$-2.306 \cdot 10^{4}$
$-2.69 \cdot 10^{4}$
$-2.098 \cdot 10^{4}$
$-4.887 \cdot 10^{3}$
$1.392 \cdot 10^{4}$
$2.534 \cdot 10^{4}$
$2.596 \cdot 10^{4}$
$1.875 \cdot 10^{4}$
$7.958 \cdot 10^{3}$
$-3.28 \cdot 10^{3}$
$-1.306 \cdot 10^{4}$
$-2.017 \cdot 10^{4}$
$-2.369 \cdot 10^{4}$



$\mathbf{Tme}(\mathbf{t}(\mathbf{tt})) =$

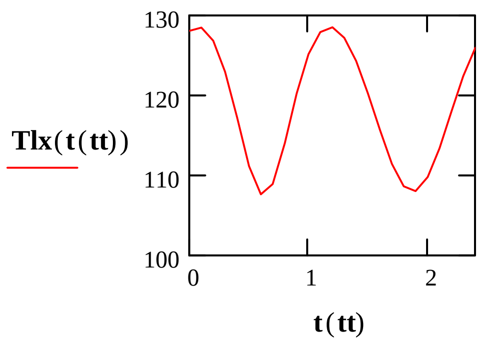
$7.182 \cdot 10^4$
$7.494 \cdot 10^4$
$7.37 \cdot 10^4$
$6.678 \cdot 10^4$
$5.305 \cdot 10^4$
$3.241 \cdot 10^4$
$6.767 \cdot 10^3$
$-2.001 \cdot 10^4$
$-4.353 \cdot 10^4$
$-6.081 \cdot 10^4$
$-7.103 \cdot 10^4$
$-7.494 \cdot 10^4$
$-7.385 \cdot 10^4$
$-6.896 \cdot 10^4$
$-6.111 \cdot 10^4$
$-5.093 \cdot 10^4$



Link Arm :

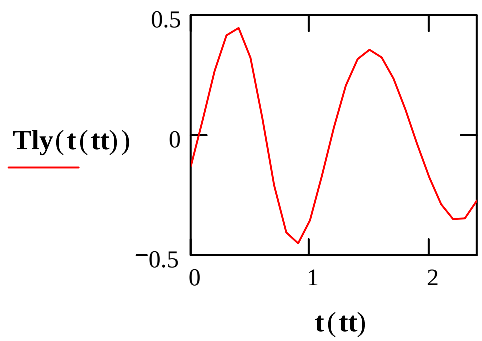
Tlx(t(tt)) =

128.116
128.505
126.841
122.96
117.22
111.179
107.676
108.921
114.079
120.261
125.175
127.946
128.55
127.239
124.33
120.251



Tly(t(tt)) =

-0.127
0.067
0.268
0.417
0.447
0.323
0.072
-0.208
-0.405
-0.45
-0.353
-0.17
0.034
0.207
0.318
0.356



INTERNAL LOADS ON BEARINGS

GVT Rotor:

$$\mathbf{Fg(t)} := \frac{\sqrt{\mathbf{Tgx(t)}^2 + \mathbf{Tgz(t)}^2}}{\mathbf{D1}}$$

Mainframe:

$$\mathbf{Fm(t)} := \left|\frac{\mathbf{Tmy(t)}}{\mathbf{D2}}\right|$$

Subframe

$$\mathbf{Fs1}(\mathbf{t}) := \frac{\mathbf{Tse}(\mathbf{t})}{\mathbf{a}}$$

$$\mathbf{Fs2}(\mathbf{t}) := \frac{-\left(\mathbf{Tlx}(\mathbf{t})\cdot\mathbf{a} + \mathbf{Tse}(\mathbf{t})\cdot\mathbf{r}\cdot\mathbf{sin}\left(\theta(\mathbf{t}) + \psi(\mathbf{t})\right)\right)}{\mathbf{a}\cdot\mathbf{r}\cdot\mathbf{cos}\left(\theta(\mathbf{t}) + \psi(\mathbf{t})\right)}$$

$$\mathrm{K}(\mathbf{t}) := \left(\left|\mathbf{Tsy}(\mathbf{t})\right| + \left|\mathbf{sin}\left(\theta(\mathbf{t}) + \psi(\mathbf{t})\right)\cdot\mathbf{Tly}(\mathbf{t}) - \mathbf{cos}\left(\theta(\mathbf{t}) + \psi(\mathbf{t})\right)\cdot\mathbf{Tlz}(\mathbf{t})\right|\right)^2$$

$$\Lambda(\mathbf{t}) := \left(\left|\mathbf{Tsz}(\mathbf{t})\right| + \left|\mathbf{cos}\left(\theta(\mathbf{t}) + \psi(\mathbf{t})\right)\cdot\mathbf{Tly}(\mathbf{t}) + \mathbf{sin}\left(\theta(\mathbf{t}) + \psi(\mathbf{t})\right)\cdot\mathbf{Tlz}(\mathbf{t})\right|\right)^2$$

$$\mathbf{Fsb}(\mathbf{t}) := \sqrt{\frac{\left(\mathrm{K}(\mathbf{t}) + \Lambda(\mathbf{t})\right)}{\mathbf{d2}}}$$

Fsb is the magnitude of the total force on subframe bearings.

Link - Arm

$$\mathbf{FL1}(\mathbf{t}) := \frac{\mathbf{Tlx}(\mathbf{t})}{\mathbf{r}}$$

$$\mathbf{FL2}(\mathbf{t}) := \frac{\mathbf{r}\cdot\mathbf{Tse}(\mathbf{t}) + \mathbf{Tlx}(\mathbf{t})\cdot\mathbf{a}\cdot\mathbf{sin}\left(\theta(\mathbf{t}) + \psi(\mathbf{t})\right)}{\mathbf{a}\cdot\mathbf{r}\cdot\mathbf{cos}\left(\theta(\mathbf{t}) + \psi(\mathbf{t})\right)}$$

Magnitude of Total Force on the top bearing:

$$\mathbf{Fltb}(\mathbf{t}) := \sqrt{\left(\left|\mathbf{FL1}(\mathbf{t})\right| + \left|\frac{\mathbf{Tlz}(\mathbf{t})}{\mathbf{d}}\right|\right)^2 + \left(\left|\mathbf{FL2}(\mathbf{t})\right| + \left|\frac{\mathbf{Tly}(\mathbf{t})}{\mathbf{d}}\right|\right)^2}$$

Magnitude of Total Force on the bottom bearing:

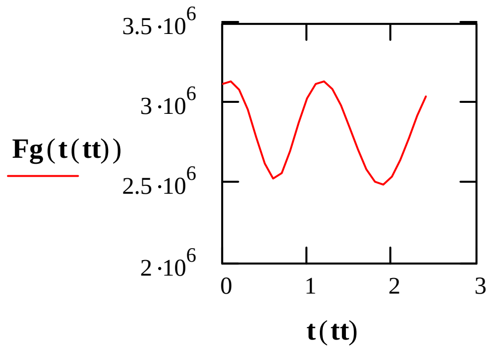
$$\mathbf{Flbb}(\mathbf{t}) := \sqrt{\left(\frac{\mathbf{Tly}(\mathbf{t})}{\mathbf{d}}\right)^2 + \left(\left|\mathbf{FL2}(\mathbf{t})\right| + \left|\frac{\mathbf{Tlz}(\mathbf{t})}{\mathbf{d}}\right|\right)^2}$$

Results:

Forces on the GVT Rotor

$\mathbf{Fg(t(tt))} =$

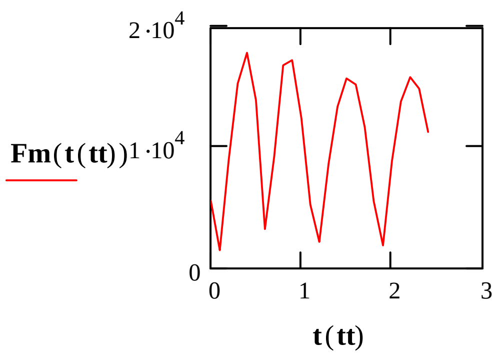
$3.124 \cdot 10^{6}$
$3.142 \cdot 10^{6}$
$3.088 \cdot 10^{6}$
$2.965 \cdot 10^{6}$
$2.794 \cdot 10^{6}$
$2.626 \cdot 10^{6}$
$2.534 \cdot 10^{6}$
$2.566 \cdot 10^{6}$
$2.705 \cdot 10^{6}$
$2.883 \cdot 10^{6}$
$3.035 \cdot 10^{6}$
$3.124 \cdot 10^{6}$
$3.142 \cdot 10^{6}$
$3.092 \cdot 10^{6}$
$2.99 \cdot 10^{6}$
$2.857 \cdot 10^{6}$



Forces on the Mainframe

$\mathbf{Fm(t(tt))} =$

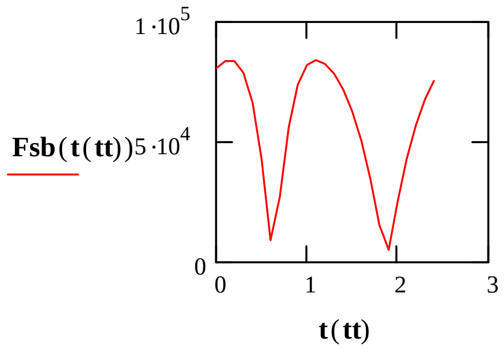
$5.625 \cdot 10^{3}$
$1.502 \cdot 10^{3}$
$9.048 \cdot 10^{3}$
$1.537 \cdot 10^{4}$
$1.794 \cdot 10^{4}$
$1.399 \cdot 10^{4}$
$3.258 \cdot 10^{3}$
$9.283 \cdot 10^{3}$
$1.69 \cdot 10^{4}$
$1.731 \cdot 10^{4}$
$1.25 \cdot 10^{4}$
$5.305 \cdot 10^{3}$
$2.187 \cdot 10^{3}$
$8.706 \cdot 10^{3}$
$1.344 \cdot 10^{4}$
$1.579 \cdot 10^{4}$



Magnitude of total force on subframe bearings

$\mathbf{Fsb(t(tt))} =$

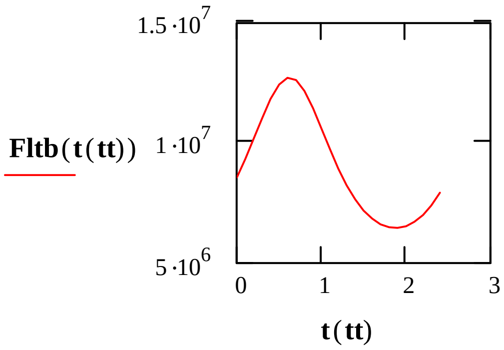
$8.085 \cdot 10^{4}$
$8.383 \cdot 10^{4}$
$8.378 \cdot 10^{4}$
$7.899 \cdot 10^{4}$
$6.651 \cdot 10^{4}$
$4.317 \cdot 10^{4}$
$9.333 \cdot 10^{3}$
$2.726 \cdot 10^{4}$
$5.632 \cdot 10^{4}$
$7.394 \cdot 10^{4}$
$8.214 \cdot 10^{4}$
$8.426 \cdot 10^{4}$
$8.265 \cdot 10^{4}$
$7.847 \cdot 10^{4}$
$7.195 \cdot 10^{4}$
$6.28 \cdot 10^{4}$



Link Arm's Total Force on the top bearing

$\mathbf{Fltb(t(tt))} =$

$8.565 \cdot 10^{6}$
$9.324 \cdot 10^{6}$
$1.017 \cdot 10^{7}$
$1.104 \cdot 10^{7}$
$1.184 \cdot 10^{7}$
$1.243 \cdot 10^{7}$
$1.272 \cdot 10^{7}$
$1.262 \cdot 10^{7}$
$1.217 \cdot 10^{7}$
$1.145 \cdot 10^{7}$
$1.061 \cdot 10^{7}$
$9.738 \cdot 10^{6}$
$8.931 \cdot 10^{6}$
$8.228 \cdot 10^{6}$
$7.644 \cdot 10^{6}$
$7.183 \cdot 10^{6}$



Link Arm's Total Force on the bottom bearing

$\mathbf{Flbb(t(tt))} =$

$8.565 \cdot 10^{6}$
$9.324 \cdot 10^{6}$
$1.017 \cdot 10^{7}$
$1.104 \cdot 10^{7}$
$1.184 \cdot 10^{7}$
$1.243 \cdot 10^{7}$
$1.272 \cdot 10^{7}$
$1.262 \cdot 10^{7}$
$1.217 \cdot 10^{7}$
$1.145 \cdot 10^{7}$
$1.061 \cdot 10^{7}$
$9.738 \cdot 10^{6}$
$8.931 \cdot 10^{6}$
$8.228 \cdot 10^{6}$
$7.644 \cdot 10^{6}$
$7.183 \cdot 10^{6}$